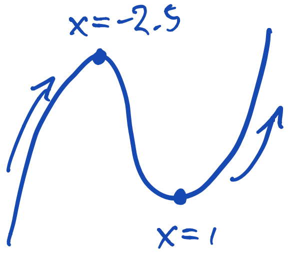
Name: Solutions PID: ____________

NOTE: You must show the steps necessary to arrive at your answer unless otherwise noted. Use your judgment, if you can't do the entire problem in your head, then you probably should write down at least some intermediate steps.

This assignment has **46 total points** and **7 pages**.

Problem 1 (4 points). Look at the graph drawn in exercise 22 of Chapter 1.3 to answer the following questions:
Warning: make sure you don't confuse the graphs from questions 22 and 24!.

(a) Describe (write down) the intervals on which the function is increasing *using interval notation*. Do not include points where the function changes from increasing to decreasing, or vice-versa.



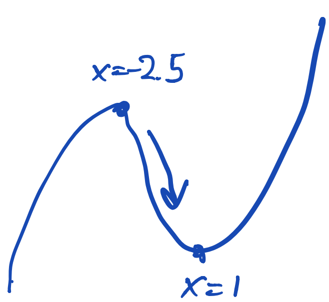
Increasing from $-\infty$ up to -2.5, and from $x=1$ off until infinity.

First interval: $(-\infty,-2.5)$

Second interval: $(1,\infty)$

Combined: $\boxed{(-\infty,-2.5)\cup(1,\infty)}$

(b) Describe (write down) the intervals on which the function is decreasing *using interval notation*. Do not include points where the function changes from increasing to decreasing, or vice-versa.

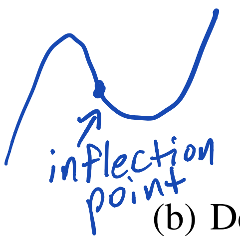


Decreasing from -2.5 to 1.

I'm not including the turnaround points, so the interval is $\boxed{(-2.5,1)}$

Problem 2 (4 points). Look at the graph drawn in exercise 34 of Chapter 1.3 to answer the following questions:

(a) Describe (write down) the intervals on which the function is concave up *using interval notation*. Do not include the inflection point.



All points from the inflection point $(x=-2)$ to ∞

Interval notation: $\boxed{(-2,\infty)}$

(b) Describe (write down) the intervals on which the function is concave down *using interval notation*. Do not include the inflection point.

All points less than the inflection point:

$\boxed{(-\infty,-2)}$

Problem 3 (2 points). Let $f(x) = 5x^2 - 2$ and $k(x) = \frac{1}{x-1}$. Evaluate $f(k(0))$ and $k(f(0))$.

$k(0) = \frac{1}{0-1} = \frac{1}{-1} = -1$

$f(0) = 5(0)^2 - 2 = 5 \cdot 0 - 2 = -2$

Therefore,

$f(k(0)) = f(-1) = 5(-1)^2 - 2 = 5 \cdot 1 - 2 = 5 - 2 = \boxed{3}$

$k(f(0)) = k(-2) = \frac{1}{-2-1} = \frac{1}{-3} = \boxed{-\frac{1}{3}}$

Problem 4 (8 points). Let $g(t) = 3\sqrt{1-t}$ and $h(t) = 1 + 4t^2$.

(a) Find the formula for $h(g(t))$.

$$h(g(t)) = h(3\sqrt{1-t}) = 1 + 4\cdot(3\sqrt{1-t})^2$$
$$= 1 + 4\cdot(3^2\cdot\sqrt{1-t}^{\,2})$$
$$= 1 + 4\cdot(9\cdot(1-t))$$
$$= 1 + 4\cdot(9-9t) = 1 + 36 - 36t = \underline{\underline{37-36t}}$$

(b) Find the formula for $g(h(t))$.

Not Graded

(c) What is the domain of $h(g(t))$?

Domain of $h(g(t))$ = Intersection of domain of g and domain of $37-36t$.

Domain of g: $(-\infty, -1]$

Domain of $37-36t$: $(-\infty, \infty)$

Intersection is $\boxed{(-\infty, 1]}$

(d) What is the domain of $g(h(t))$?

Not Graded

Problem 5 (4 points). For each of the following, describe whether the function $h(n)$ is shifted up, down, left, and/or right **and** by how many units:

(a) $h(n + 32)$

Shifted left by 32 units

(b) $-5 + h(n)$

Shifted down by 5 units

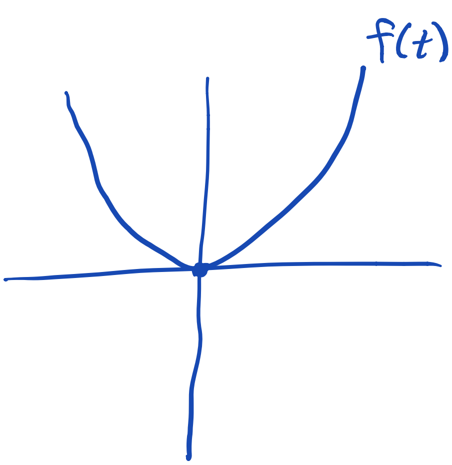
(c) $h(n - 7)$

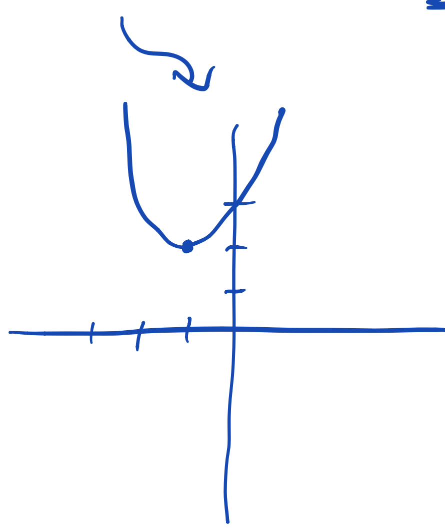
Shifted right by 7 units

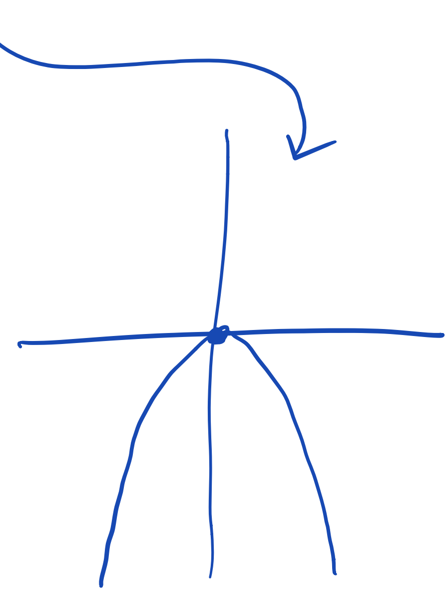
(d) $h(n + 2) - 2$

shifted left by 2 units AND down by 2 units

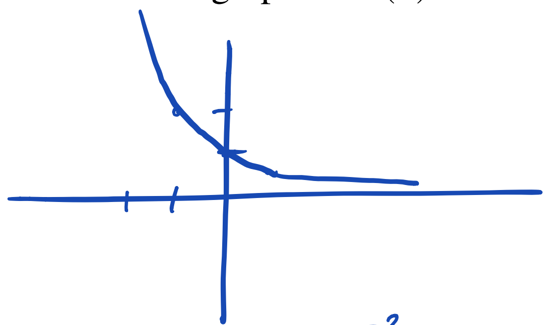
Problem 6 (4 points). Sketch the graph of $f(t) = t^2$. Then, sketch the graph of $f(t)$ shifted up by 2 units and left by 1 unit. Finally, sketch $-f(t)$. Label each graph and your axes.







Problem 7 (2 points). Look at the graph of $f(x) = 2^x$ drawn for Exercise 32 in Chapter 1.5. Sketch below the graph for $h(x) = 2^{-x}$.



Horizontal reflection of original graph

Problem 8 (2 points). Is $g(z) = 4z^4 - \overset{z^2}{\cancel{x^2}} + 2$ an even function, odd function, or neither? Justify briefly.

g is an even function:

$g(-z) = 4(-z)^4 - (-z)^2 + 2 = 4z^4 - z^2 + 2 = g(z)$

Problem 9 (2 points). Is $d(s) = \sqrt[3]{s}$ an even function, odd function, or neither? Justify briefly.

d is an odd function, since

$d(-s) = \sqrt[3]{-s} = -\sqrt[3]{s} = -d(s)$

Problem 10 (4 points). Write a formula to describe the following:

(a) The absolute value function vertically stretched by a factor of 2 and horizontally stretched by a factor of 3.

absolute value function is $f(x) = |x|$

Vertical stretch: $2 \cdot f(x)$

Horizontal + Vertical stretch: $2 \cdot f(\frac{1}{3}x) = 2|\frac{1}{3}x|$

(b) The reciprocal function reflected over the y axis.

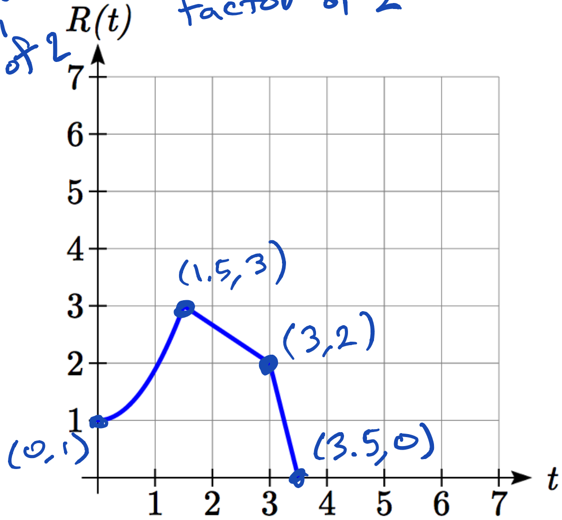
Reciprocal function: $f(x) = \frac{1}{x}$

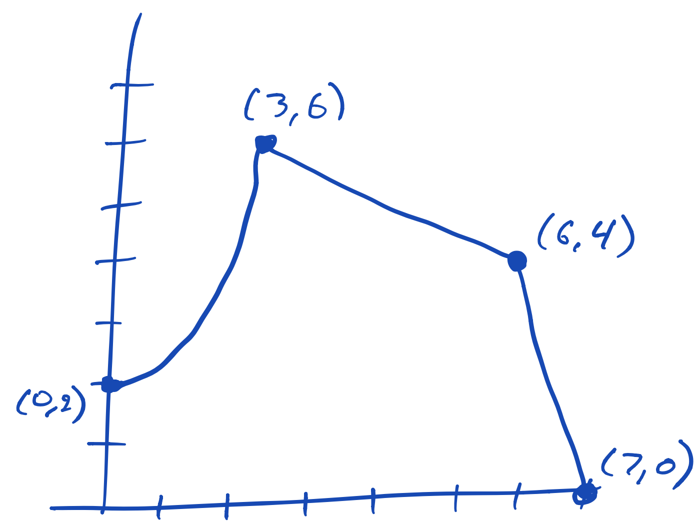
Reflection over y-axis: $f(-x) = \frac{1}{-x} = -\frac{1}{x}$

Problem 11 (4 points). Consider the graph of $R(t)$ below. Sketch the graph of $2 \cdot R\left(\frac{1}{2}t\right)$ on the interval $[0, 7]$.

Vertical stretch factor of 2

Horizontal stretch Factor of 2





Problem 12 (1 point). Let $f(x)$ be some one-to-one function. If $f(4) = 5$, what is $f^{-1}(5)$?

$f^{-1}(5) = 4$

Problem 13 (1 point). Consider the function $h(z)$ defined by the following table. Create a corresponding table for $h^{-1}(z)$.

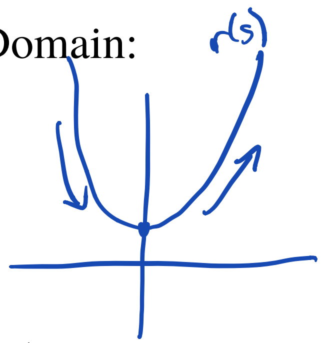
z	-2	10	6	1	2
h(z)	1.5	2	2.5	6	8

z	1.5	2	2.5	6	8
$h^{-1}(z)$	-2	10	6	1	2

Problem 14 (4 points). Let $r(s) = 2s^2 + 1$. Find a domain on which r is one-to-one and non-decreasing. Then, determine the formula for r^{-1} on that domain.

Extra Credit

Domain:



Non-decreasing for $s \geq 0$

and one-to-one here as well

$r^{-1}(s) =$

Just chose a random letter name

Let $t = r(s) = 2s^2+1$

Then $t-1 = 2s^2$

$\Rightarrow \frac{t-1}{2} = s^2$

$\Rightarrow \pm\sqrt{\frac{t-1}{2}} = s$

Now, I chose my domain to be $s \geq 0$ above, so I don't consider anything where $s<0$.

Therefore I drop the negative case here, since that would make $s<0$.

So formula is $\boxed{r^{-1}(t) = \sqrt{\frac{t-1}{2}}}$

[OPTIONAL]
Survey Question.
1. Roughly how many hours did you spend working on this homework assignment?
2. Do you find the lectures to go too fast, too slow, or roughly the right speed?