Name: Solutions PID: ____________

NOTE: You must show the steps necessary to arrive at your answer unless otherwise noted. Use your judgment, if you can't do the entire problem in your head, then you probably should write down at least some intermediate steps.

This assignment has 12 pages. There are 48 total points.

Problem 1 (8 points). Consider the function $f(x) = (0.4)^x$.

(a) What is the domain of $f(x)$?

All real numbers

(b) What is the range of $f(x)$?

All $y > 0$

(c) Determine a formula for the inverse of $f(x)$.

Solve for x: $y = (0.4)^x \rightarrow \log_{(0.4)}(y) = x \rightarrow f(x) = \log_{0.4}(x)$

(d) Does the graph of $f(x)$ have any asymptotes? Where?

Yes, horizontal asymptote at $y = 0$.

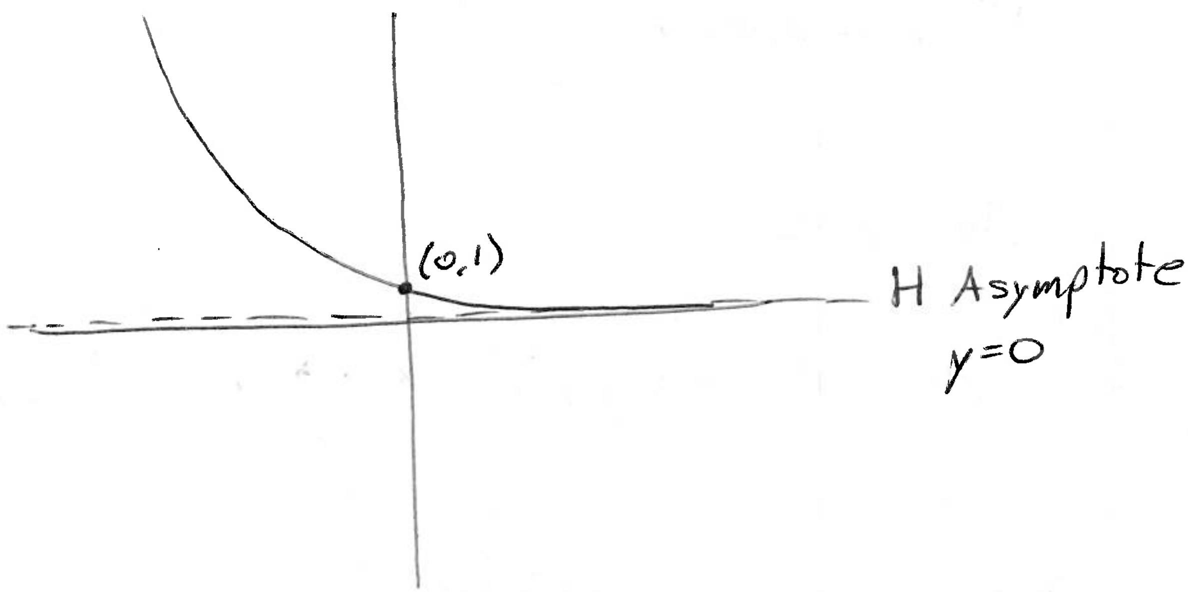
(e) Does the graph of $f(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

~~No~~ Yes, at (0, 1)

(f) Does the graph of $f(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

No

(g) Sketch the graph of $f(x)$. Label any intercepts or asymptotes.



Your answers need not be this wordy

Problem 2 (8 points). Consider the function $g(x) = -2(0.4)^{x+3}$.

(a) What is the domain of $g(x)$?

H Shift left by 3, shifts domain of $f(x)$ left by 3 → Domain of $f(x)$ was all real numbers, so shifted left, domain of $g(x)$ is all real numbers

(b) What is the range of $g(x)$?

$g(x)$ is $f(x)$ stretched vertically by power of 2 and reflected vertically.

Range of $f(x)$: $(0, \infty)$

Range of $g(x)$: $(-\infty, 0)$

(c) Determine a formula for the inverse of $g(x)$.

Solve for x: $y = -2(0.4)^{x+3} \rightarrow \frac{-y}{2} = 0.4^{x+3}$

$\rightarrow \log_{0.4}\left(\frac{-y}{2}\right) = x+3 \rightarrow x = \log_{0.4}\left(\frac{-y}{2}\right) - 3 \rightarrow g^{-1}(x) = \log_{0.4}\left(\frac{-x}{2}\right) - 3$

(d) Does the graph of $g(x)$ have any asymptotes? Where?

Yes, ~~vertical~~ horizontal asymptote at $y=0$

(e) Does the graph of $g(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

Yes $g(0) = -2(0.4)^{0+3} = -2(0.4)^3$

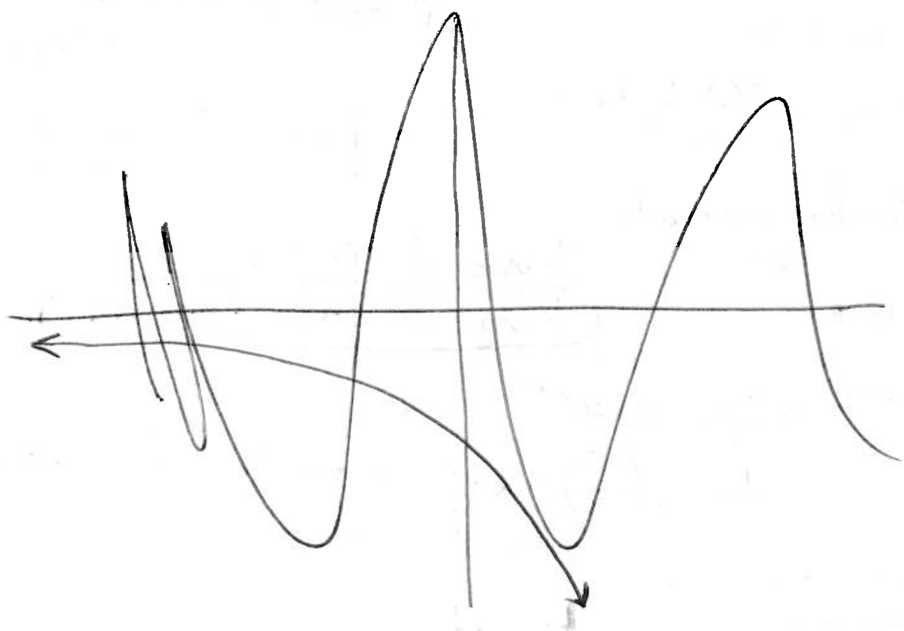
Intercept $= (0, -2(0.4)^3)$

No simplification needed

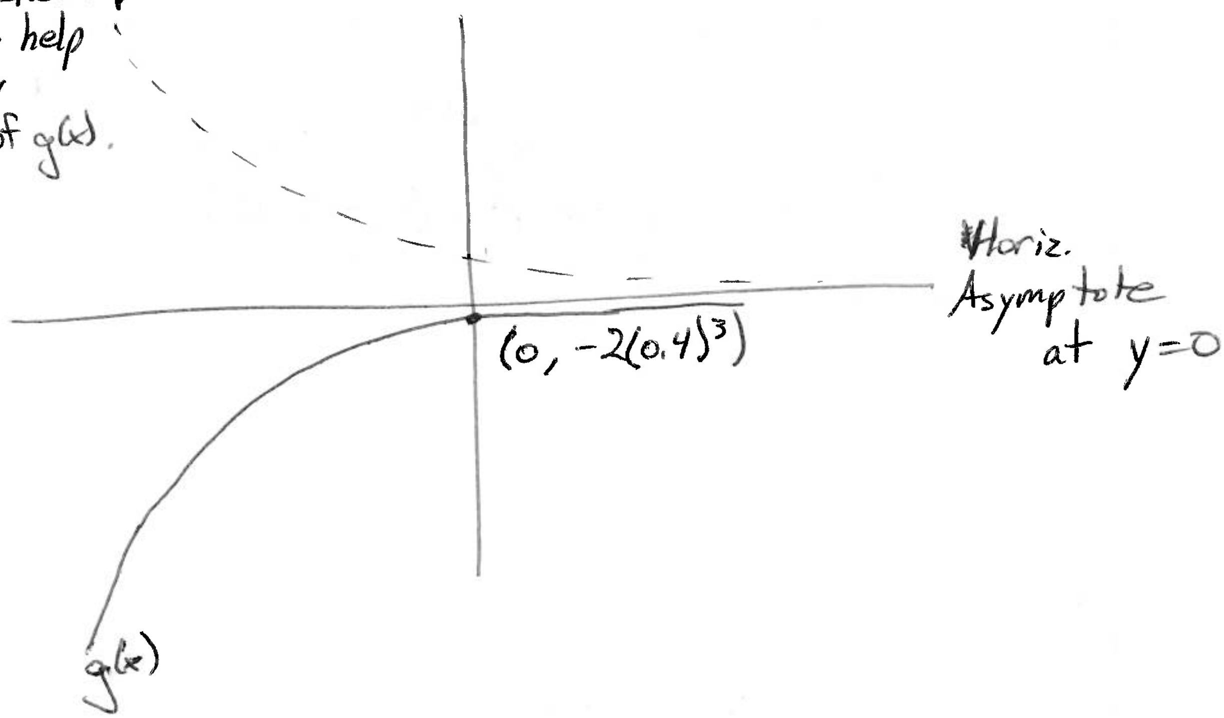
(f) Does the graph of $g(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

No

(g) Sketch the graph of $g(x)$. Label any intercepts or asymptotes.



This is my sketch of f(x) to help me in my sketch of g(x).



Problem 3 (8 points). Consider the function $h(x) = \log_6(x)$.

(a) What is the domain of $h(x)$?

$(0, \infty)$

(b) What is the range of $h(x)$?

All real numbers

(c) Determine a formula for the inverse of $h(x)$.

Solve for x: $y = \log_6(x) \longrightarrow x = 6^y \longrightarrow h^{-1}(x) = 6^x$

(d) Does the graph of $h(x)$ have any asymptotes? Where?

Yes, vertical asymptote at $x = 0$

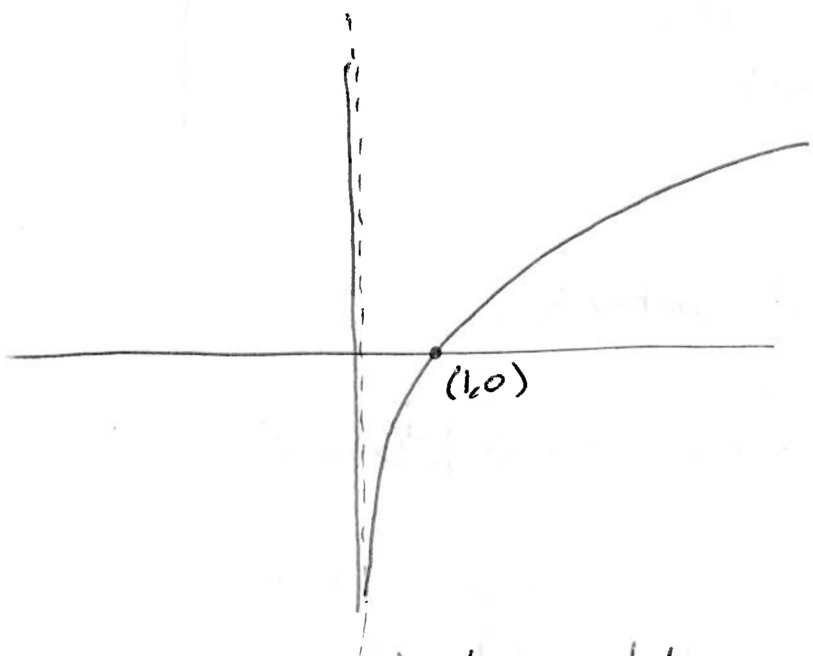
(e) Does the graph of $h(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

No

(f) Does the graph of $h(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

$(1, 0)$

(g) Sketch the graph of $h(x)$. Label any intercepts or asymptotes.



vertical asymptote
at $x=0$

Problem 4 (8 points). Consider the function $k(x) = 2\log_6(x-2)+4$.

(a) What is the domain of $k(x)$?

$k(x)$ is like $h(x)$, but shifted ~~hor~~ right by 2 and vertically stretched/shifted

Domain of $h(x)$ was $(0,\infty)$, so domain of $k(x)$ is $\boxed{(2,\infty).}$

(b) What is the range of $k(x)$?

$k(x)$ is like $h(x)$, but vertically shifted up by 4 and vertically stretched by 2, and horizontally shifted

Since range of $h(x)$ was $(-\infty,\infty)$, range of $k(x)$ is also $\boxed{(-\infty,\infty)}$

(c) Determine a formula for the inverse of $k(x)$.

Solve for x: $y = 2\log_6(x-2)+4$

$\Rightarrow \frac{y-4}{2} = \log_6(x-2)$ $\to 6^{\frac{y-4}{2}} = x-2$ $\to x = 6^{\frac{y-4}{2}}+2$ $\to$ $\underline{k^{-1}(x) = 6^{\frac{x-4}{2}}+2}$

(d) Does the graph of $k(x)$ have any asymptotes? Where?

Yes, vertical asymptote at $\underline{x=2}$ ← This is the asymptote of $h(x)$ shifted right by 2

(e) Does the graph of $k(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

~~[illegible]~~ No

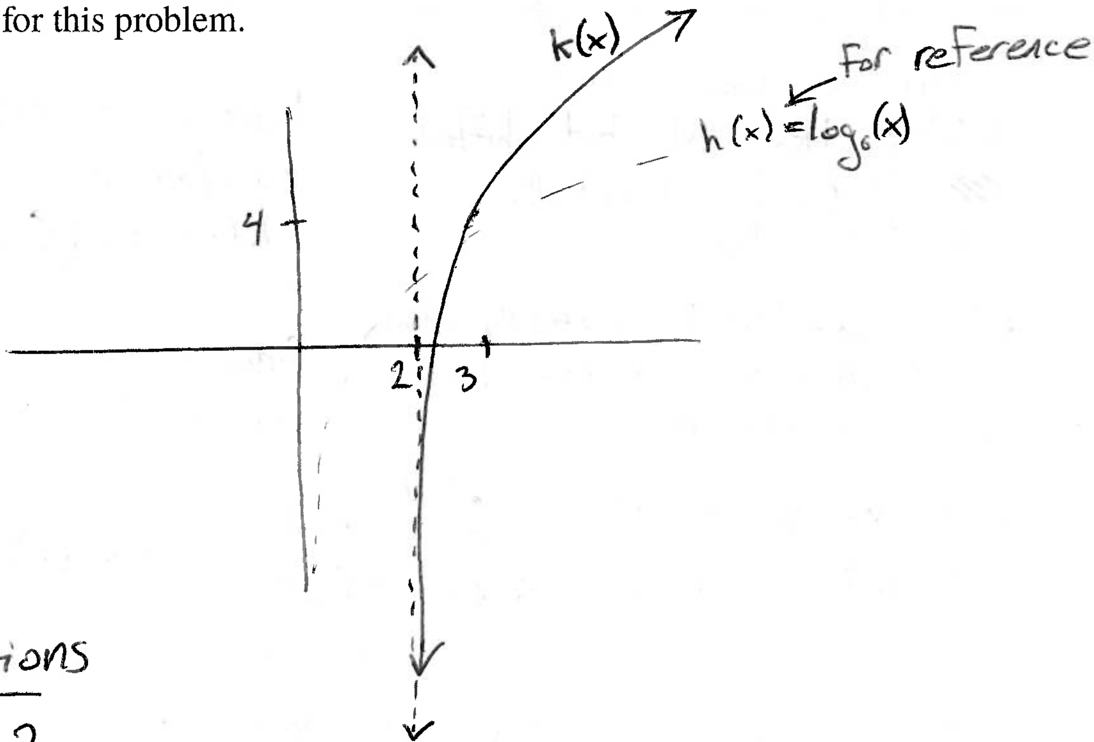
(f) Does the graph of $k(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

~~[illegible]~~

$\boxed{\text{Not Graded}}$

Sorry!

(g) Sketch the graph of $k(x)$. Label asymptotes, but you do *not* need to label intercepts for this problem.



Transformations

Shift right 2

Stretch vertically by 2

Shift up 4

Problem 5 (8 points). Consider the function $s(x) = \cos(x)$.

(a) What is the domain of $s(x)$?

All real numbers

(b) What is the range of $s(x)$?

$[-1, 1]$

(c) Determine a formula for the inverse of $s(x)$ (this is not a trick question, see Chapter 6.3).

$\cos^{-1}(x)$ OR $s^{-1}(x)$ does not exist

if domain restricted to $[0, \pi]$

(d) Does the graph of $s(x)$ have any asymptotes? Where?

No

(e) Does the graph of $s(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

Yes. $s(0) = \cos(0) = 1 \longrightarrow (0, 1)$

(f) Does the graph of $s(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

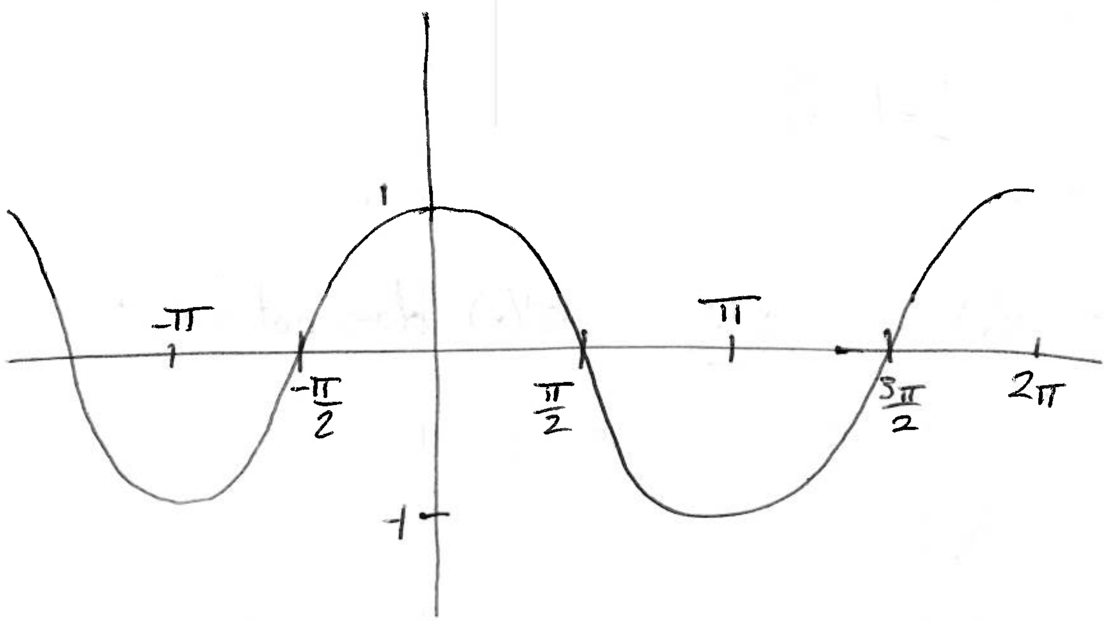
Yes. Find all points such that $\cos(x) = 0$.

$\rightarrow x = \frac{\pi}{2} + 2\pi \cdot n$

$\rightarrow x = \frac{3\pi}{2} + 2\pi \cdot n$

where n is an integer

(g) Sketch the graph of $s(x)$. Label intercepts or asymptotes in a clear way.



Problem 6 (8 points). Consider the function $r(x) = -\cos(3x) + 5$.

(a) What is the domain of $r(x)$?

All real numbers

(b) What is the range of $r(x)$?

$[4, 6]$ ← range of $s(x)$ shifted up by 5

Will not be assessed on final

(c) Determine a formula for the inverse of $r(x)$.

No inverse exists

OR

On the interval $[0, \frac{2\pi}{3}]$, $r^{-1}(x) = \cos^{-1}(5-x) \cdot \frac{1}{3}$

(d) Does the graph of $r(x)$ have any asymptotes? Where?

No

(e) Does the graph of $r(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

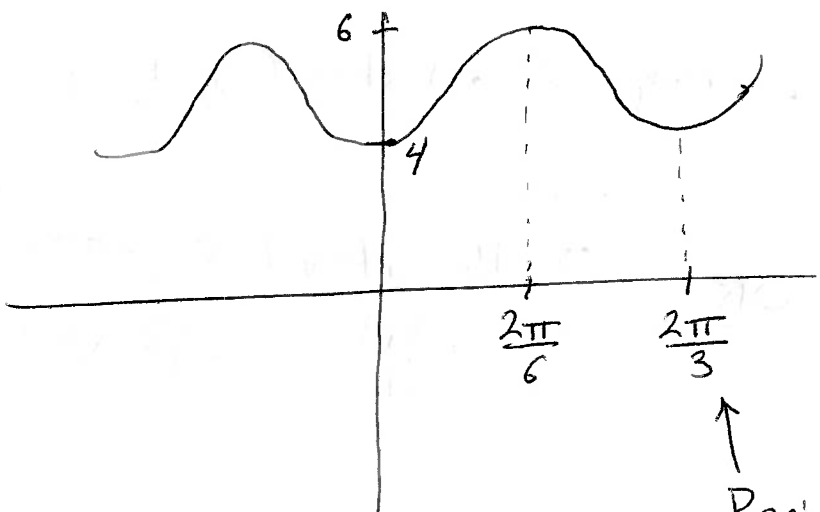
Yes. $r(0) = -\cos(3 \cdot 0) + 5 = -\cos(0) + 5 = -1 + 5 = 4$

$\Rightarrow (0, 4)$

(f) Does the graph of $r(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

No. Range is $[4, 6]$, so $r(x)$ never touches horizontal axis.

(g) Sketch the graph of $r(x)$. Label intercepts or asymptotes in a clear way.



Period is $\frac{2\pi}{3}$ because $r(x)$ has a horizontal stretch by $\frac{1}{3}$