Practice Midterm 2

Math 3C: Precalculus
Instructor: David Lenz

Midterm: Thursday November 21 at 8:00 PM in Ledden Auditorium.
Bring your student ID. Do NOT bring a calculator or any formula sheets.

Problem 1 True or False. Write the word "True" or "False" next to each statement. You *do not* need to show your work for this question.

True — The function $k(x) = \log_5(x)$ is one-to-one.

False — The equation $(x+1)^2 + (x-4)^2 = 4$ represents a circle of radius 4 centered at the point $(-1, 4)$.

True — An angle with measure $30°$ is coterminal with an angle of measure $-690°$.

False — The function $3a^3 - 2a^2 + 10a + 2$ has at most 4 horizontal intercepts.

False — The function $g(z) = \frac{z+1}{z+2}$ has a vertical asymptote at $z = -1$.

True — An angle that measures $\frac{3\pi}{4}$ radians is $135°$ when measured in degrees.

False — The range of $y = 2^{x-2}$ is all $y > 2$.

True — $r(s) = 2 + 4s^2 - s^6 + 3s^4$ is an even function.

multiplicity = 2

multiplicity = 1

Problem 2 Let $p(y) = -2(y-2)^2(y+1) = -2y^3 + 6y^2 - 8$.

What is the long-run behavior of $p(y)$?

Matches long-run behavior of leading term

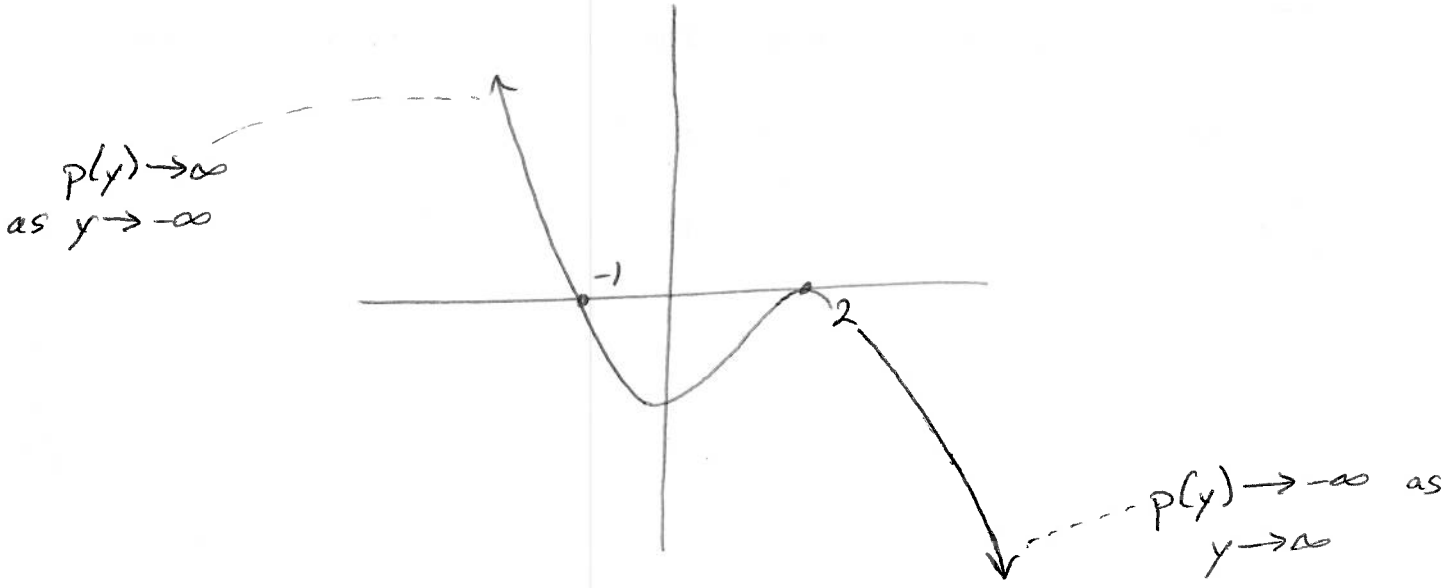
As $y \to \infty$, $p(y) \to -\infty$

As $y \to -\infty$, $p(y) \to \infty$

What are the roots of $p(y)$ and their multiplicities?

Roots:	Multiplicities:
$y = -1$	1
$y = 2$	2

Sketch a graph of $p(y)$. Your sketch needs to only show the general shape of the graph, but you must label the horizontal intercepts.



Problem 3 Solve the equation $3 \cdot (5)^{4x-1} = 12$ for x.

$3 \cdot 5^{4x-1} = 12$

$5^{4x-1} = 4$

$\log_5(5^{4x-1}) = \log_5(4)$

$4x - 1 = \log_5(4)$

$4x = \log_5(4) + 1$

$x = \frac{\log_5(4)+1}{4}$

Problem 4 Solve the equation $\log_3(9a^4) = 3$ for a.

$\log_3(9a^4) = 3$

$3^3 = 9a^4$ by definition of logs

$27 = 9a^4$

$3 = a^4$

$a = \sqrt[4]{3}$

(i.e. $a = 3^{\frac{1}{4}}$)

$\log_3(9) + \log_3(a^4) = 3$

$2 + \log_3(a^4) = 3$

$\log_3(a^4) = 1$

$4 \cdot \log_3(a) = 1$

$\log_3(a) = \frac{1}{4}$

$3^{\frac{1}{4}} = a$

(i.e. $\sqrt[4]{3} = a$)

Two Methods

Problem 5 Suppose that there is a bank account with \$10000 dollars in it that earns interest at an annual rate of 10%. How much money will be in the account after 2 years?

Annual rate of 10% → +10% each year

After 1 year: $10000 + (10\% \text{ of } 10{,}000)$
$= 10000 + (0.10 \cdot 10000)$
$= 10000 + 1000 = 11000$

After 2 years: $11000 + (10\% \text{ of } 11000)$
$= 11000 + (0.10 \cdot 11000)$
$= 11000 + 1100$
$= 12100$

Let $C(t)$ be a function that gives the amount of money in the account after t years. What is a formula for $C(t)$?

$$\boxed{C(t) = 10000 \cdot 1.1^t}$$

Equivalently,

$$C(t) = 10000(1 + .1)^t$$

Problem 6 Solve the equation $2y^2 - 6y - 3 = \frac{1}{2}$ for y.

$$2y^2-6y-3=\frac{1}{2}$$

$$\Rightarrow 2y^2-6y-3-\frac{1}{2}=0$$

$$\Rightarrow 2y^2-6y-\frac{7}{2}=0$$

$a \quad b \quad c$

By the quadratic equation,

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(\frac{-7}{2})}}{2(2)}$$

$$y=\frac{6\pm\sqrt{36-4(2)(\frac{-7}{2})}}{4}$$

$$y=\frac{6\pm\sqrt{36+28}}{4}$$

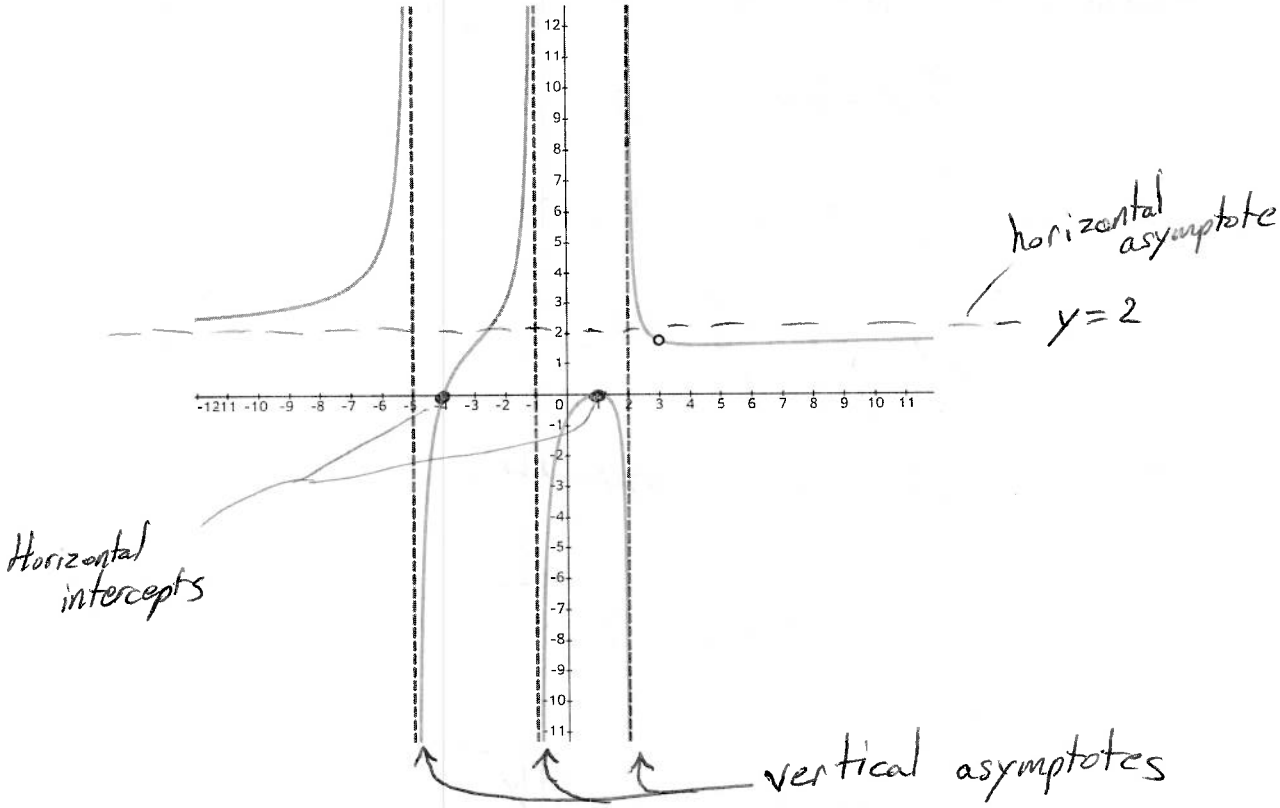
$$y=\frac{6\pm\sqrt{64}}{4}$$

$$y=\frac{6\pm 8}{4}=\frac{6+8}{4} \text{ or } \frac{6-8}{4}$$

$$y=\frac{14}{4} \text{ or } \frac{-2}{4}$$

$$y=\frac{7}{2} \text{ or } \frac{-1}{2}$$

Problem 7 Let $g(b)$ be the function graphed in the following diagram.



What is the horizontal asymptote of $g(b)$?

$y=2$

What are the vertical asymptotes $g(b)$?

$b=2$, $b=-1$, and $b=-5$

What are the horizontal intercepts of $g(b)$?

$b = -4$ and $b = 1$

Which of the following could be a formula for $g(b)$? *(circle one)*

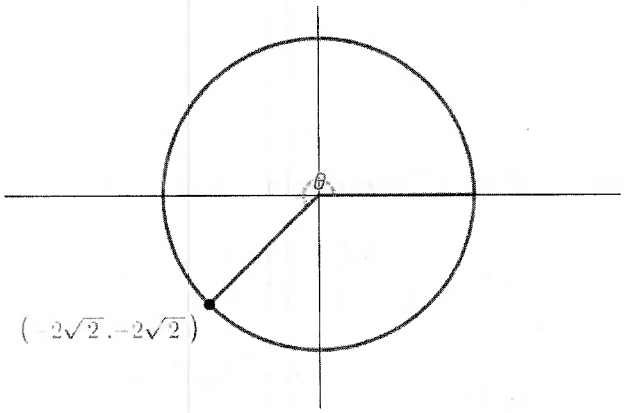
$$g(b) = \frac{2(b-1)^2(b+4)}{(b+1)(b+5)(b-2)}$$ X No hole at $b = 3$

$$g(b) = \frac{(b-3)(b+2)(b-5)}{(b+5)(b-1)(b-3)}$$ X Horizontal intercepts are $b=-2$ and $b=5$

$$g(b) = \frac{2(b-3)(b-1)^2(b+4)}{(b+1)(b+5)(b-2)(b-3)}$$

$$g(b) = \frac{2(b+1)(b+5)(b-2)(b-3)}{(b-1)^2(b+4)(b-3)}$$ X Vertical asymptotes at $b=1$ and $b=-4$

Problem 8 Let θ be the angle in the following picture. The circle pictured has radius 4.



What is $\sin(\theta)$?

$$\sin(\theta) = \frac{y}{r} = \frac{-2\sqrt{2}}{4} = \frac{-\sqrt{2}}{2}$$

What is $\cos(\theta)$?

$$\cos(\theta) = \frac{x}{r} = \frac{-2\sqrt{2}}{4} = \frac{-\sqrt{2}}{2}$$