Practice Midterm 1

Math 3C: Precalculus

Instructor: David Lenz

Midterm: Thursday October 24 at 8:00 PM in Ledden Auditorium.
Bring your student ID. Do NOT bring a calculator or any formula sheets.

Problem 1 Consider two quantities, u and v, which are related to each other with the following table:

x	4	-2	0	1	6	7	3
f(x)	4	2	3	6	8	4	2

x	2	6	4	1	8	0	3
g(x)	3	2	5	6	-1	4	1

Determine the following:

(a) What is $f(g(0))$?

$g(0) = 4$

$f(g(0)) = f(4) = \underline{4}$

(b) What is $g(f(0))$?

$f(0) = 3$

$g(f(0)) = g(3) = \underline{1}$

(c) What is $f(f(1))$?

$f(1) = 6$

$f(f(1)) = f(6) = \underline{8}$

(d) What is $f(g^{-1}(1))$?

$g(3) = 1$, so $g^{-1}(1) = 3$.

Hence $f(g^{-1}(1)) = f(3) = \underline{2}$

Problem 2 Determine where the lines described by $a(x) = \frac{3}{4}x - 2$ and $b(x) = \frac{-5}{4}x + 4$ intersect and write your answer as a coordinate pair. Are these two lines perpendicular?

$$a(x) = b(x)$$
$$\frac{3}{4}x - 2 = \frac{-5}{4}x + 4$$
$$\frac{3}{4}x = \frac{-5}{4}x + 6$$
$$\frac{3}{4}x + \frac{5}{4}x = 6$$
$$\frac{8}{4}x = 6$$
$$2x = 6$$
$$x = 3$$
$$a(3) = \frac{3}{4}\cdot 3 - 2 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$$

Intersects at $\boxed{(3, \frac{1}{4})}$

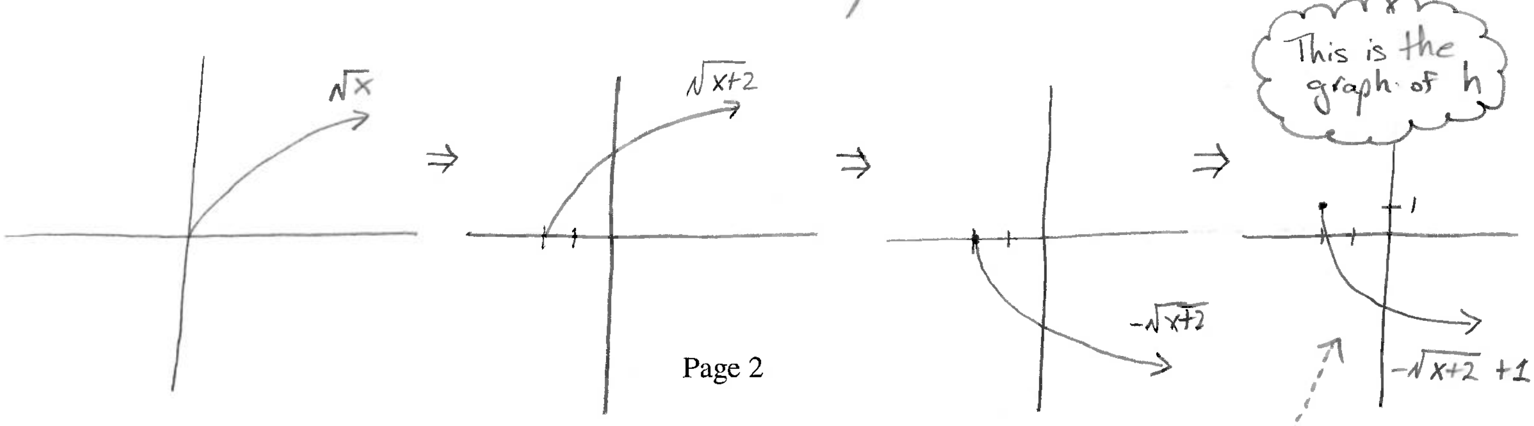
Slope of $a = \frac{3}{4}$, slope of $b = \frac{-5}{4}$.

$\frac{3}{4} \cdot \frac{-5}{4} = \frac{-15}{16} \neq -1$, so lines are <u>not</u> <u>perpendicular</u>

Problem 3 What is the range of $\cancel{h(z)}$ $h(x)$ $= -\sqrt{x+2}+1$? Write your answer in inequality notation.

Range of $h(\cancel{z})$ is the set of all possible outputs of ~~h(z)~~ $h(x)$.

$h(x) = \cancel{h(z)} = -\sqrt{x+2} + 1$ is like $\sqrt{x}$, but it is shifted horizontally, reflected, and shifted vertically.



The outputs are all less than or equal to -1, so range is all $x \leq 1$

I didn't intend for this problem to be quite so hard. You should understand the idea, but midterm won't be as messy

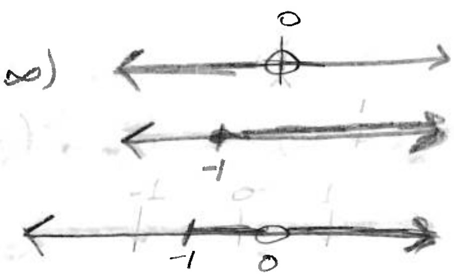
(d) What is the domain of $(h \circ g)(s)$? Write your answer in inequality or interval notation.

The domain of $h \circ g$ is the intersection of the domain of g and the domain of the formula in (c), which is: $\frac{|2s+1|}{s}$.

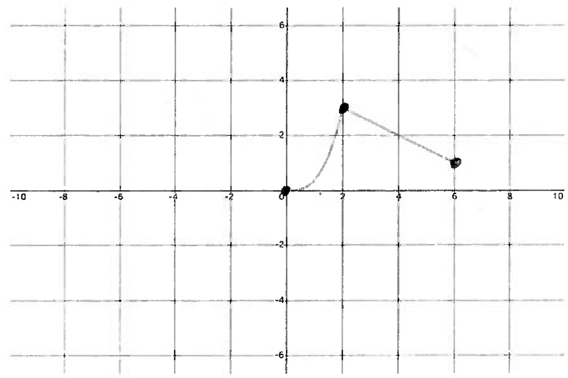
Domain of this is all $s \neq 0$, or $(-\infty, 0) \cup (0, \infty)$

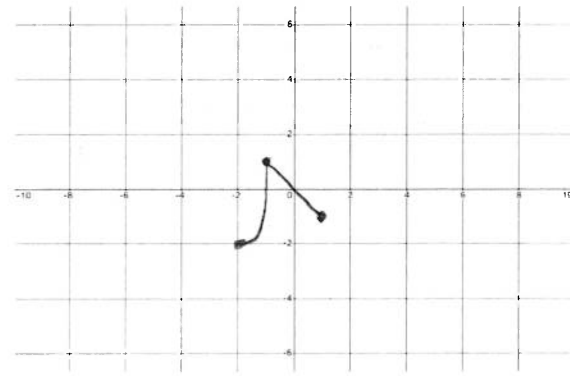
Domain of g is $[-1, \infty)$

Intersection of these $[-1, 0) \cup (0, \infty)$



Problem 5 Consider the graph of $r(k)$ below. Sketch the graph of $r(2k + 4) - 2$.





$r(2k+4)-2$

Left shift by 4

H. Compression by $\frac{1}{2}$

Shift down by 2

Left Shift, Horiz Compress, Down Shift

$(0,0) \longrightarrow (-4,0) \longrightarrow (-2,0) \longrightarrow (-2,-2)$

$(2,3) \longrightarrow (-2,3) \longrightarrow (-1,3) \longrightarrow (-1,1)$

$(6,1) \longrightarrow (2,1) \longrightarrow (1,1) \longrightarrow (1,-1)$

Problem 4 Let $h(s) = \frac{|2s^2-1|}{s^2-1}$ and $g(s) = \sqrt{s+1}$.

(a) What is the domain of $h(s)$? Write your answer in inequality or interval notation.

Domain is the set of all possible inputs to $h(s)$.
Can't divide by 0, so $s^2-1 \neq 0$. This means $s^2 \neq 1$, which means $s \neq 1$ and $s \neq -1$.

$\boxed{\text{Interval notation: } (-\infty,-1)\cup(-1,1)\cup(1,\infty)}$ $\boxed{\text{Inequality: } s<-1 \text{ or } -1<s<1 \text{ or } s>1}$

(b) What is the domain of $g(s)$? Write your answer in inequality or interval notation.

All possible inputs to $g(s)$. Can't take the square root of a negative number, so $s+1$ must be greater than or equal to -1.

$\boxed{\text{Interval: } [-1,\infty)}$ $\boxed{\text{Inequality: } s \geq -1}$

(c) Determine a formula for $(h \circ g)(s)$.

$$(h\circ g)(s) = h(g(s)) = h(\sqrt{s+1})$$
$$= \frac{|2(\sqrt{s+1})^2-1|}{(\sqrt{s+1})^2-1}$$
$$= \frac{|2(s+1)-1|}{(s+1)-1}$$
$$= \frac{|2s+2-1|}{s}$$
$$= \frac{|2s+1|}{s}$$

Problem 6 Is $f(t) = \frac{3}{t^2-1}$ a one-to-one function? Why or why not?

<u>No,</u> because $f(2) = \frac{3}{2^2-1} = \frac{3}{3} = 1$

and $f(-2) = \frac{3}{(-2)^2-1} = \frac{3}{3} = 1$

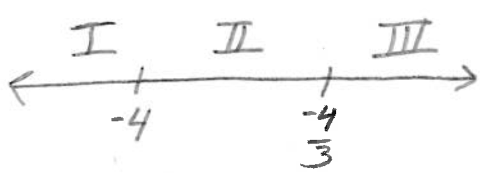
So this function has an output with more than one corresponding input, which makes it not one-to-one.

Problem 7 Solve the inequality $|3x + 8| < 4$ for x. Write your answer in inequality and interval notation.

Solve equality first: $|3x+8| = 4$

Two Cases:

$3x+8=4$ | $-(3x+8)=4$
$3x=-4$ | $-3x-8=4$
$x=\frac{-4}{3}$ | $-3x=12$
 | $x=-4$



Point Test: $x=-5$. $|3\cdot(-5)+8| = |-15+8| = |-7| = 7 \leftarrow$ <u>not</u> <4 ✗

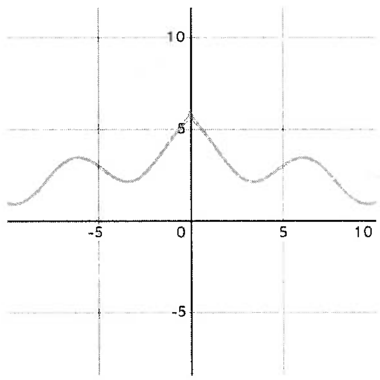
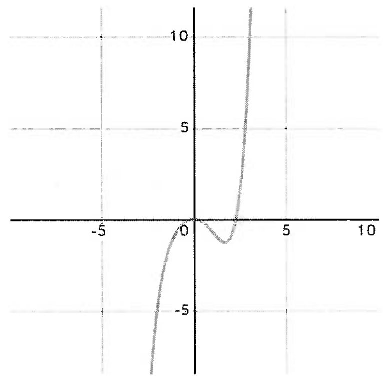
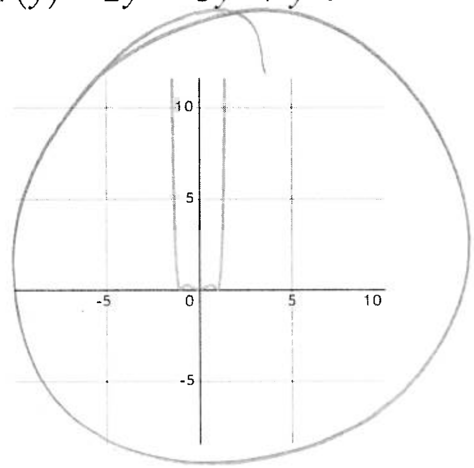
$x=-3$. $|3\cdot(-3)+8| = |-9+8| = |-1| = 1 \leftarrow$ yes <4 ✓

$x=0$. $|3\cdot 0+8| = |8| = 8 \leftarrow$ <u>not</u> <4 ✗

So only region II passes the point test. Since the question was a strict inequality $<$, endpoints are <u>not</u> included.

$\boxed{(-4, \frac{-4}{3})}$ $\boxed{-4 < x < \frac{-4}{3}}$

Problem 8 Which of the following could be the graph of $r(y) = 2y^8 - 3y^6 + y^2$?

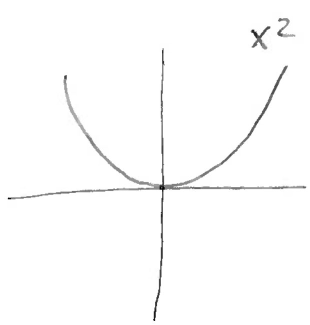




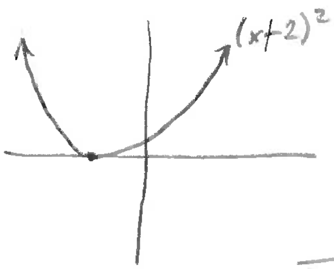
Problem 9 Let $f(x) = -(x+2)^2 - 3$

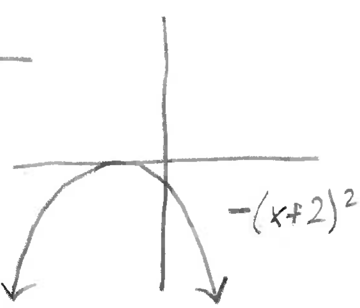
(a) Sketch $f(x)$

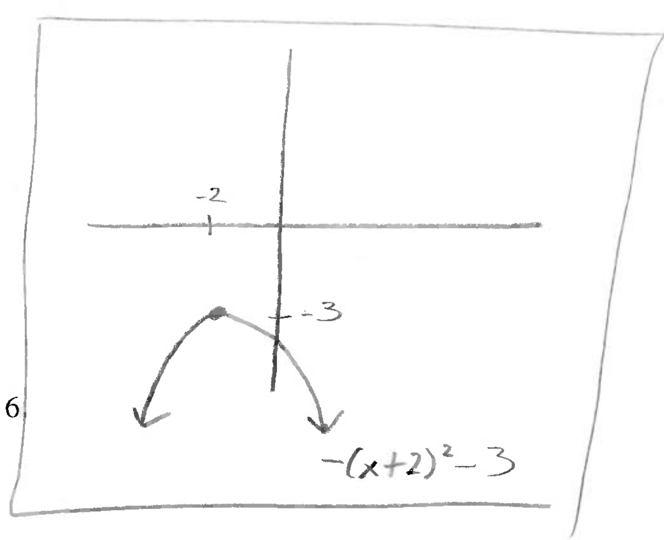
This is like $f(x) = x^2$, shifted left by 2, shifted down by 3, and vertically reflected

When combining vertical transformation, we do reflections before shifts









(b) Describe the region where f is non-decreasing using inequality notation

Non-decreasing from $-\infty$ to -2 (inclusive)

That is, all $\boxed{x \leq -2}$

(c) Restricting the domain of f to the region in part (b), determine the formula for $f^{-1}(x)$.

$$f(x) = -(x+2)^2 - 3$$

$$y = -(x+2)^2 - 3$$

Solve for x: $y + 3 = -(x+2)^2$

$$-y - 3 = (x+2)^2$$

$$\pm\sqrt{-y-3} = x + 2$$

$$\pm\sqrt{-y-3} - 2 = x$$

Now, since we restrict x to be $x \leq -2$, we <u>exclude</u> the case where $\sqrt{-y-3} - 2 = x$, since this could make $x > -2$.

Hence $-\sqrt{-y-3} - 2 = x$

or $\boxed{f^{-1}(y) = -\sqrt{-y-3} - 2}$