Practice Final

Math 3C: Precalculus

Instructor: David Lenz

Final: Monday December 9 at 8:00 AM in YORK 2622.
Bring your student ID. Do NOT bring a calculator or any formula sheets.

Problem 1 True or False. Write the word "True" or "False" next to each statement. You *do not* need to show your work for this question.

True The function $f(z) = 2|z|$ is even.

False If $t(a) = a$, then $t^{-1}(a) = \frac{1}{a}$.

False The arc spanned by a 270° angle on a circle of radius 2 is 2π units long.

False The range of $\cos^{-1}(y)$ is $[\frac{-\pi}{2}, \frac{\pi}{2}]$.

True The long-run behavior of $p(x) = 3x^2 - 4x - 5x^5 + 2$ is that $p(x) \to \infty$ as $x \to -\infty$ and $p(x) \to -\infty$ as $x \to \infty$.

False An angle that measures $\frac{7\pi}{4}$ radians is 270° when measured in degrees.

False The function $k(b) = \frac{b}{b^2-1}$ has no horizontal asymptotes.

True The line passing through the points $(2, 4)$ and $(7, 8)$ has a slope of $\frac{4}{5}$.

True $\sin(3x)$ is an odd function.

True $g(y) = 2y^4$ is not one-to-one.

False The circle centered at $(1, -1)$ with radius 4 is described by the equation $(x + 1)^2 + (x - 1)^2 = 16$.

Problem 2 True or False. Write the word "True" or "False" next to each statement. You *do not* need to show your work for this question.

False A bank account with \$1000 and an annual interest rate of 4% will generate more money if interest is compounded monthly than if interest is compounded daily.

False For some numbers a, b, and c, with $a, b > 0$, $\log_b(a^c) = \log_b(a) + c$.

True The number e is less than 3.

$(b \circ a)(x) = e^{3x} + 3x$

False If $a(x) = 3x$ and $b(x) = e^x + x$, then $(b \circ a)(x) = e^{3x} + x$.

True The total number of students enrolled at UCSD is a function of the year.

Problem 3 Solve the equation $3|2 + 4x| = 12$ for x.

$$3|2+4x| = 12$$

$$\Rightarrow |2+4x| = 4$$

$$2+4x = 4 \qquad\qquad -(2+4x) = 4$$

$$4x = 2 \qquad\qquad 2+4x = -4$$

$$4x = -6$$

$$x = \frac{1}{2} \qquad\qquad x = \frac{-3}{2}$$

Problem 4 Find the points where the circle described by $x^2+(y-3)^2 = 15$ intersects the line $y = 2x + 3$. Write your answer as a coordinate pair(s); your answer may involve radicals that can't be simplified.

$$x^2+(y-3)^2=15$$

and $$y=2x+3$$

Substituting:

$$x^2+((2x+3)-3)^2=15$$

$$\Rightarrow x^2+(2x)^2=15$$

$$\Rightarrow x^2+4x^2=15$$

$$\Rightarrow 5x^2=15$$

$$\Rightarrow x^2=3$$

$$\Rightarrow x=\pm\sqrt{3}$$

Plug back into equation:

For $x=\sqrt{3}$:

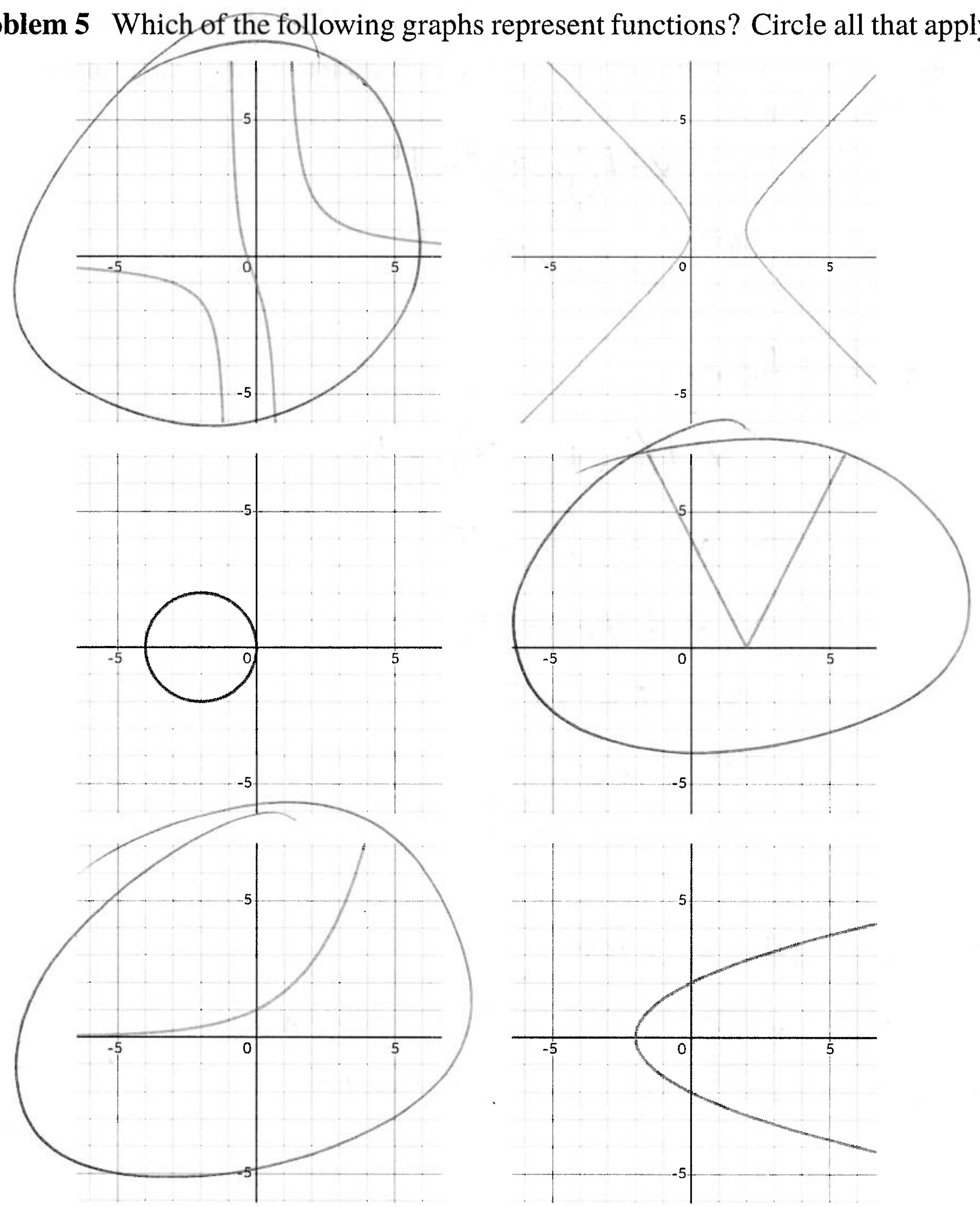
$y=2x+3 \rightarrow \underline{y=2\sqrt{3}+3}$

For $x=-\sqrt{3}$

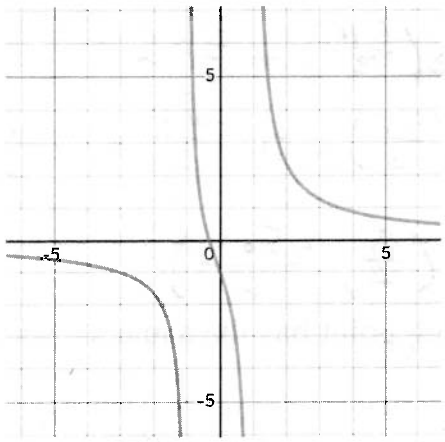
$y=2x+3 \rightarrow y=-2\sqrt{3}+3$

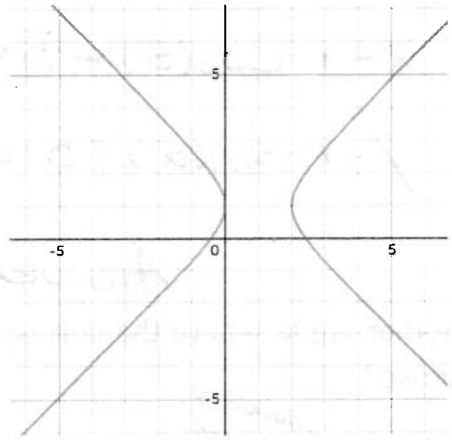
The two intersection points are $(\sqrt{3}, 2\sqrt{3}+3)$ and $(-\sqrt{3}, -2\sqrt{3}+3)$

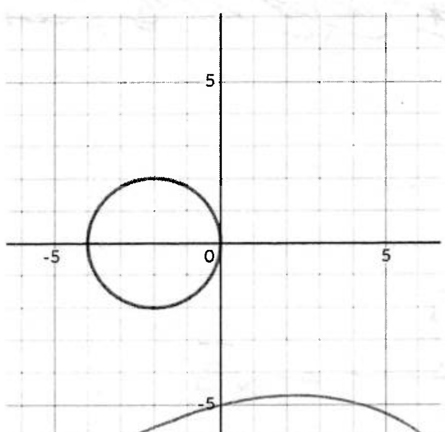
Problem 5 Which of the following graphs represent functions? Circle all that apply.

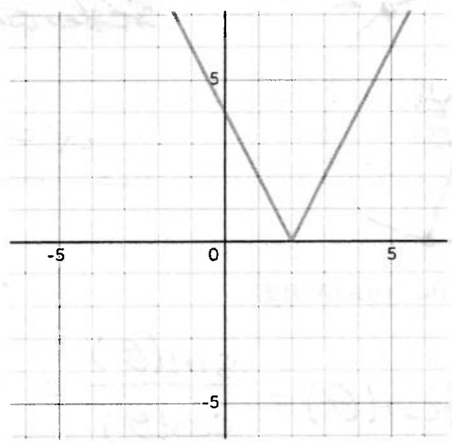


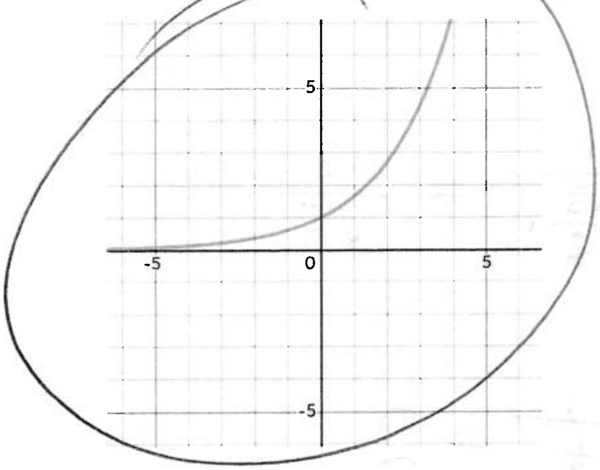
Problem 6 Which of the following graphs represent **one-to-one** functions? Circle all that apply.

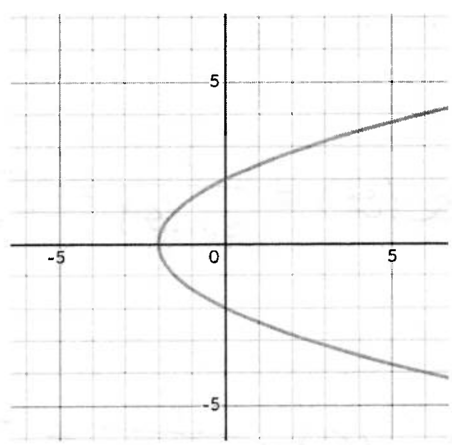




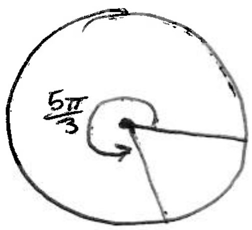








Problem 7 What are the coordinates of the point on a circle (centered at the origin) of radius 5 at an angle of $\theta = \frac{5\pi}{3}$ radians?

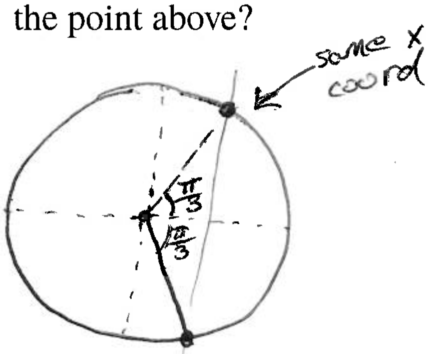


$$x = r\cdot\cos(\theta) = 5\cdot\cos\left(\frac{5\pi}{3}\right) = 5\cdot\frac{1}{2} = \frac{5}{2}$$

$$y = r\cdot\sin(\theta) = 5\cdot\sin\left(\frac{5\pi}{3}\right) = 5\cdot\frac{-\sqrt{3}}{2} = \frac{-5\sqrt{3}}{2}$$

Answer: $\left(\frac{5}{2}, \frac{-5\sqrt{3}}{2}\right)$

What is another angle where the corresponding point has the same x-coordinate as the point above?



~~Reference angle is $\frac{\pi}{3}$ for~~

$$\alpha = \frac{\pi}{3}$$

Compute the following:

tan(θ)

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

csc(θ)

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{-\sqrt{3}}{2}} = \frac{-2}{\sqrt{3}}$$

sec(θ)

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\frac{1}{2}} = 2$$

Problem 8 Let $p(x) = -2x(x-3)^2$.

What is the long-run behavior of $p(x)$?

$-2x(x-3)^2 = -2x(x^2-6x+9) = -2x^3+12x^2-18x$, so

$p(x) \longrightarrow \infty$ as $x \longrightarrow -\infty$ and

$p(x) \longrightarrow -\infty$ as $x \longrightarrow \infty$

What is the **vertical** intercept of $p(x)$?

$p(0) = -2\cdot 0 \cdot (0-3)^2 = 0$.

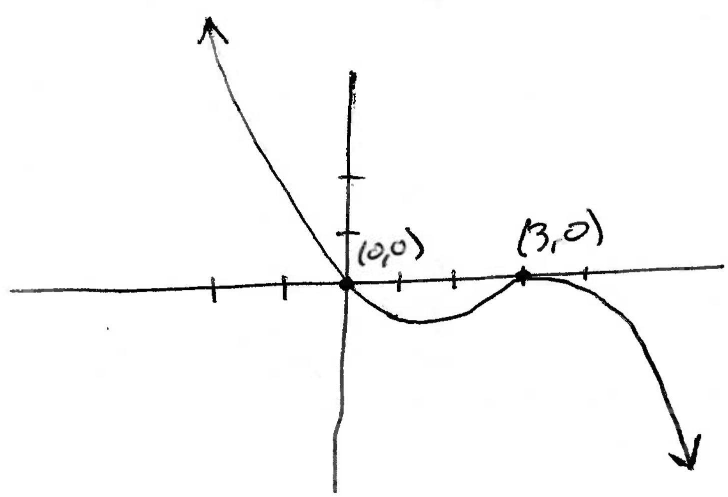
Vertical intercept at $(0,0)$.

What are the **horizontal** intercepts(s) of $p(x)$, and what are their multiplicites?

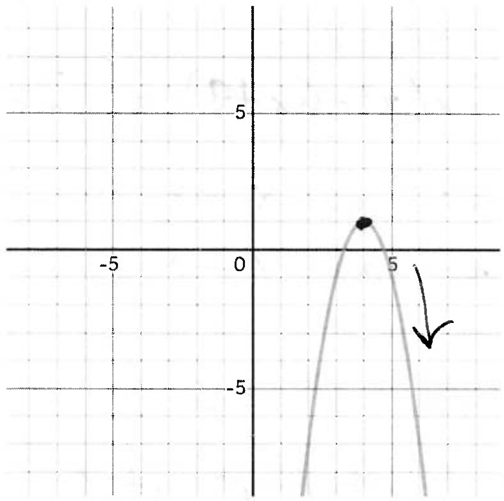
$x=0$ and $x=3$

multiplicity: 1 multiplicity: 3

Sketch a graph of $p(x)$. Use descriptive tick marks and label all intercepts.



Problem 9 Which of the following formulas could represent the parabola shown in the diagram?



Vertex at (4, 1)

Opens downward

(a) $f(x) = -(x+4)^2 - 1$ X Wrong vertex

(b) $g(x) = (x+1)^2 - 4$ X Wrong vertex

(c) $h(x) = -2(x-4)^2 + 1$

(d) $k(x) = 3(x-4)^2 + 1$ X Opens up, not down

(e) $l(x) = -4(x-1)^2 + 4$ X Wrong vertex

Problem 10 Solve the equation $2 \cdot 5^{y-4} = 14$ for y. Your answer may have exponentials or logarithms that cannot be simplified.

$$2 \cdot 5^{y-4} = 14$$

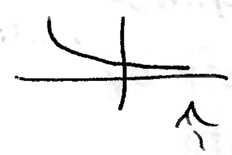
$$\Rightarrow \quad 5^{y-4} = 7$$

$$\Rightarrow \quad \log_5(5^{y-4}) = \log_5(7)$$

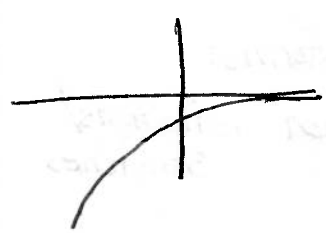
$$\Rightarrow \quad y-4 = \log_5(7)$$

$$\Rightarrow \quad \boxed{y = 4 + \log_5(7)}$$

Problem 11 Is the function $q(y) = -2 \cdot (0.5)^y$ increasing or decreasing?

$(0.5)^y$ is decreasing,  (exponential decay)

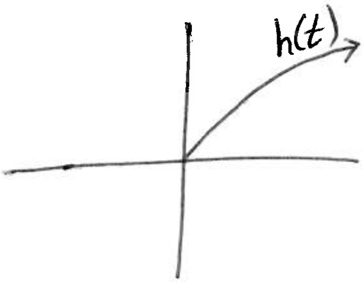
$-2 \cdot (0.5)^y$ is a vertical reflection and stretch of this

so it looks like: 

So function $q(y)$

is increasing.

Problem 12 What is the domain and range of $h(t) = \sqrt{t}$?



Domain: $[0, \infty)$

Range: $[0, \infty)$

Let $r(t) = 2\sqrt{3-t} + 4$. $r(t)$ can be obtained from $h(t)$ through a series of four transformations. What are these transformations?

$2\sqrt{3-t}\ +4$

vertical shift up by 4 units

vertical stretch by a factor of 2

Horizontal shift left by 3 units

Horizontal reflection

What are the domain and range of $r(t)$?

Domain is affected by horizontal transforms:

- Shift left by 3 units
- Horiz. Reflect

Order of H. Transforms:
First shifts, then reflections/stretches.

Therefore new domain is

$[0,\infty) \to [-3,\infty) \to (-\infty, 3]$

old Domain

shift left by 3

reflect

Range is affected by vertical transforms:

- Shift up by 4 units
- Stretch vertically by factor of 2

Order of V. Transforms:
First reflections/stretches, then shifts.

Therefore, new range is

$[0,\infty) \to [0,\infty) \to [4,\infty)$

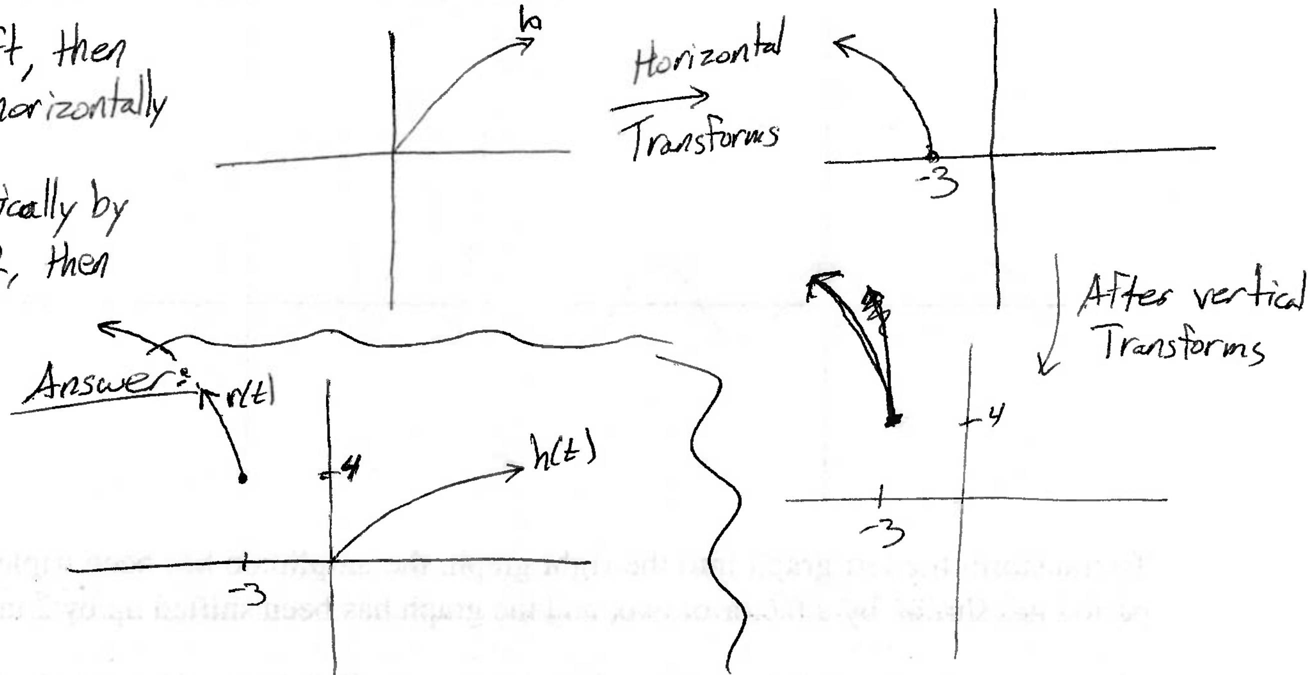
multiply by 2

add 4

Sketch $r(t)$ and $h(t)$ on the same set of axes. Your sketch doesn't need to be exact but should reflect the transformations described above.

Shift left, then reflect horizontally

Stretch vertically by factor of 2, then shift up



Problem 13 Find the point where the two lines $u(x) = 3x + 1$ and $v(x) = 6x - 2$ intersect. Write your answer as a coordinate pair.

Set $u(x) = v(x)$

$3x + 1 = 6x - 2$

$\Rightarrow 3x = 6x - 3$

$\Rightarrow -3x = -3$

$\Rightarrow x = 1$

Plug into equation:

$u(x) = 3(1) + 1$

$= 3 + 1$

$= 4$

$\Rightarrow$ Intersect at $(1, 4)$

Consider two new lines: $f(c) = 2c - 5$ and $g(c) = \frac{1}{2}c + 3$. Are the lines $f(c)$ and $g(c)$ perpendicular? Why or why not?

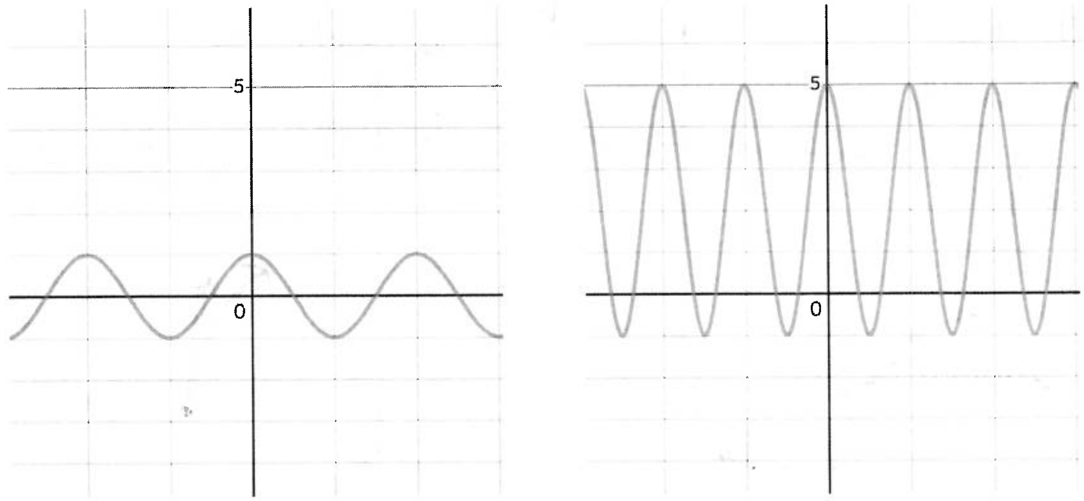
Not perpendicular, because two lines are perpendicular when their slopes multiply to -1.

Slope of f: 2

~~slop~~ Slope of g: $\frac{1}{2}$

$\longrightarrow$ Product $= 2 \cdot \frac{1}{2} = 1$, not -1

Problem 14 Consider the two graphs below.



To transform the left graph into the right graph, the amplitude has been tripled, the period has shrunk by a factor of two, and the graph has been shifted up by 2 units.

The formula for the graph on the left is $\cos(\pi \cdot x)$. Determine the formula for the graph on the right.

Amplitude tripled $\longrightarrow$ Vertical stretch by factor of 3

Period shrunk by factor of 2 $\longrightarrow$ Horizontal stretch by factor of $\frac{1}{2}$

Shifted up by 2 $\longrightarrow$ Vertical shift up by 2

New Formula: $\boxed{\underbrace{3}_{\text{V Stretch}}\cos(\underbrace{2}_{\text{H Stretch}}\pi x) + \underbrace{2}_{\text{V Shift}}}$

Problem 15 Find the zeros of the function $y(a) = \frac{2(a-12)(a-3)(a+4)}{(a+3)(a-3)}$.

$y(a) = 0$ when numerator is 0 AND denominator is not zero

numerator $= 0$ if $a = 12$, $a = 3$, or $a = -4$

denominator $= 0$ if $a = -3$ or $a = 3$

So $y(a) = 0$ when $\boxed{a = 12}$ or $\boxed{a = -4}$

Problem 16 Let $g(s) = \ln(s+2)$ and $h(s) = e^s$. Compute and simplify $(h \circ g)(3)$.

$$(h \circ g)(3) = h(g(3)) = h(\ln(3+2))$$
$$= h(\ln(5))$$
$$= e^{\ln(5)}$$
$$= 5$$

Problem 17 Use trigonometric identities to simplify the following expressions:

$(1 - \cos^2(x))\csc(x)$

$1-\cos^2(x) = \sin^2(x)$, so

$$(1-\cos^2(x))\csc(x) = \sin^2(x)\csc(x)$$
$$= \sin^2(x) \cdot \frac{1}{\sin(x)} = \underline{\sin(x)}$$

$\frac{1}{2\csc(x)sec(x)}$

$$\frac{1}{2\csc(x)\sec(x)} = \frac{1}{2 \cdot \frac{1}{\sin(x)} \cdot \frac{1}{\cos(x)}} = \frac{1}{2}\sin(x)\cos(x)$$

Now, $\sin(2x) = 2\sin(x)\cos(x)$, so

$$= \frac{1}{4} \cdot 2\sin(x)\cos(x)$$
$$= \boxed{\frac{1}{4} \cdot \sin(2x)}$$

$\sec(x)(\cos(x) + \tan(x)\sin(x))$

$$\sec(x)(\cos(x) + \tan(x)\sin(x))$$
$$= \frac{1}{\cos(x)}\left(\cos(x) + \frac{\sin(x)}{\cos(x)} \cdot \sin(x)\right)$$
$$= \frac{1}{\cos(x)}\left(\cos(x) + \frac{\sin^2(x)}{\cos(x)}\right)$$
$$= \frac{\cos(x)}{\cos(x)} + \frac{\sin^2(x)}{\cos(x)\cos(x)} = 1 + \frac{\sin^2(x)}{\cos^2(x)}$$
$$= 1 + \tan^2(x)$$
$$= \underline{\underline{\sec^2(x)}}$$

Problem 18 Using the fact that $4^{1.5} = 8$, simplify the following expression.

$$\frac{2\log_4(6) + \log_4(2) - \log_4(9)}{\log_4(16)}$$

If $4^{1.5} = 8$, then

$\log_4(8) = 1.5$

Now, $\dfrac{2\log_4(6) + \log_4(2) - \log_4(9)}{\log_4(16)}$

subtraction property and power property

$= \dfrac{\log_4(6^2) + \log_4\left(\frac{2}{9}\right)}{\log_4(16)}$

$= \dfrac{\log_4(36) + \log_4\left(\frac{2}{9}\right)}{\log_4(16)}$

addition property

$= \dfrac{\log_4\left(36 \cdot \frac{2}{9}\right)}{\log_4(16)}$

$36 \cdot \frac{2}{9} = \frac{72}{9} = 8$

$= \dfrac{\log_4(8)}{\log_4(16)} = \dfrac{1.5}{\log_4(16)} = \dfrac{1.5}{2} = 0.75$

since $4^2 = 16$,
$\log_4(16) = 2$