

Midterm 1

Math 3C: Precalculus
Instructor: David Lenz
October 24, 2019

Name: Solutions PID: _____

Seat Number: _____

Do not begin this exam until instructed to do so.

When the exam begins, **FIRST** write your name and PID at the top of each page. **Pages without a name may not be graded.**

There are 8 problems on this exam. You must show the steps you took to arrive at your answer in order to receive full points (unless explicitly stated otherwise). If you need more space for your answer than is provided, write a clear statement that your answer continues on a separate page. Then, using an empty sheet of scratch paper, continue your answer and submit this sheet with your exam. You must tell the TA that you have extra pages when you turn in your exam. If you need to repeat this process for multiple questions, use a separate sheet for each question.

The use of calculators and formula sheets (“cheat sheets”) is prohibited on this exam. All phones and similar electronic devices, as well as notes and notebooks must be put away in a closed bag or likewise out of sight.

At the end of the exam, bring your student ID and exam to the front of the hall to be collected.

NAME:

PID:

Midterm 1

Problem 1 True or False. Write the word “True” or “False” next to each statement. You *do not* need to show your work for this question.

False The function $k(x) = 2x^2 - 1$ is one-to-one.

True Two lines are parallel if they have the same slope.

False Two lines are perpendicular if they have different slopes.

False If $f(x)$ is some function, then $f(5x)$ transforms f with a horizontal stretch (expansion) by a factor of 5.

False The domain of a function is the set of all possible outputs of that function.

True If $f(x)$ is some function, then the graph of $f(x - 1)$ is the same as the graph of $f(x)$, but shifted right by one unit.

False The set $(5, 8]$ contains the number 5.

True $f(x) = |x|$ is an even function.

Problem 2 Let $m(y) = 6y - 5$ and $n(y) = -3y + 1$. Where do the graphs of $m(y)$ and $n(y)$ intersect? Write your answer as a coordinate pair.

$$m(y) = n(y)$$

$$6y - 5 = -3y + 1$$

$$6y = -3y + 6$$

$$9y = 6$$

$$y = \frac{2}{3}$$

$$n\left(\frac{2}{3}\right) = -3 \cdot \frac{2}{3} + 1$$

$$= -2 + 1$$

$$= -1$$

Intersects at $\left(\frac{2}{3}, -1\right)$

Is $m(y)$ an increasing function?

Yes, slope is positive

Is $n(y)$ an increasing function?

No, slope is negative

Problem 3 Let $f(x)$ and $g(x)$ be defined by the following tables. You do not need to show your work for this question.

x	6	-2	2	3	5	-1	4
f(x)	4	1	3	8	0	-1	2

x	2	6	4	1	8	0	3
g(x)	1	2	3	4	-1	-5	1

Determine the following:

What is the domain of f ? You can write your answer as a list.

$$6, -2, 2, 3, 5, -1, 4$$

What is the range of f ? You can write your answer as a list.

$$4, 1, 3, 8, 0, -1, 2$$

Is $g(x)$ a one-to-one function? Why or why not?

No, $g(2) = 1$ and $g(3) = 1$, so not every output has a unique input.

What is $(g \circ f)(4)$?

$$(g \circ f)(4) = g(f(4)) = g(2) = 1$$

What is $f^{-1}(3)$?

$$f(4) = 3, \text{ so } f^{-1}(3) = 4$$

Problem 4 Solve the equation $|2x - 8| = 6$ for x .

If $|2x - 8| = 6$, then

either $(2x - 8) = 6$ or $-(2x - 8) = 6$

↙
 $2x = 14$

$$\underline{x = 7}$$

↓
 $-2x + 8 = 6$

$$-2x = -2$$

$$\underline{x = 1}$$

Problem 5 Let

$$a(z) = \sqrt[3]{2z-3}.$$

Find a formula for $a^{-1}(z)$.

$$\text{Let } y = a(z) = \sqrt[3]{2z-3}$$

Solve for z :

$$y = \sqrt[3]{2z-3}$$

$$y^3 = 2z - 3$$

$$y^3 - 3 = 2z$$

$$\frac{y^3 - 3}{2} = z$$

$$\text{So } a^{-1}(y) = \frac{y^3 - 3}{2}$$

Equivalently, I can rename the variable and
say

$$a^{-1}(z) = \frac{z^3 - 3}{2}$$

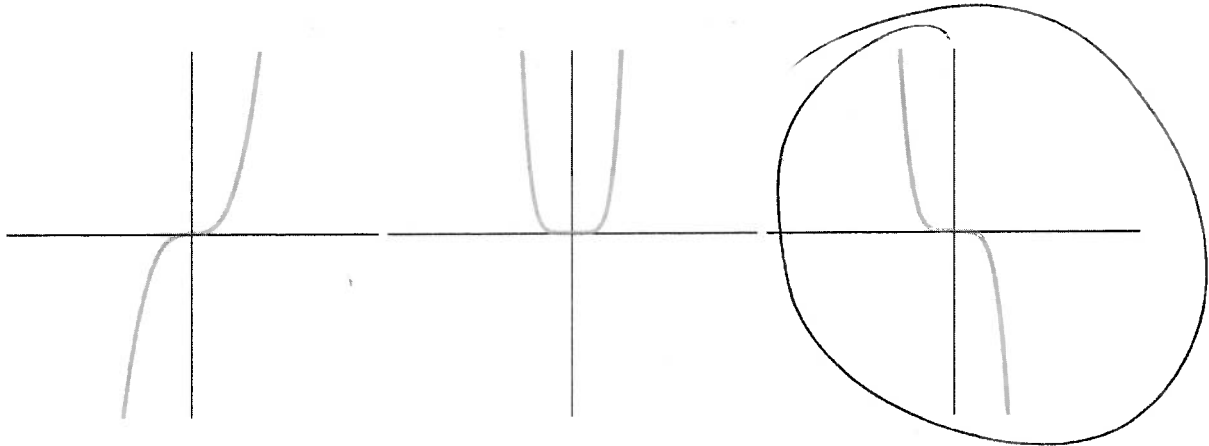
NAME:

PID:

Midterm 1

Problem 6 Suppose $h(t)$ is a function, and the long-run behavior of h is that $h(t)$ tends to ∞ as $t \rightarrow -\infty$, and that $h(t)$ tends to $-\infty$ as $t \rightarrow \infty$.

Which of the following graphs could possibly represent $h(t)$? (circle one)



Which of the following formulas could be the formula for $h(t)$? (circle one)

$$h(t) = -t^6$$

$$h(t) = -t^5$$

$$h(t) = t^3$$

$$h(t) = t^2$$

NAME:

PID:

Midterm 1

Problem 7 Let $f(x) = \sqrt{1 + x^2}$ and $g(x) = \sqrt{3x - 1}$. Find a formula for $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(\sqrt{3x-1}) \\ &= \sqrt{1 + (\sqrt{3x-1})^2} \\ &= \sqrt{1 + (3x-1)} \\ &= \sqrt{3x} \end{aligned}$$

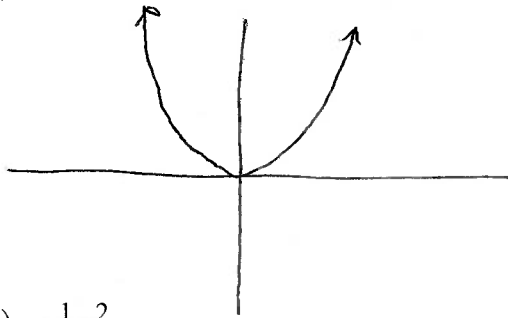
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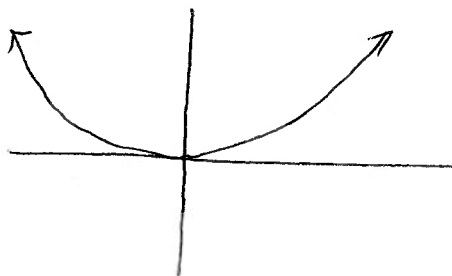
Midterm 1

Problem 8 Sketch the graphs of the following functions. Each function is a slight modification of the function before it.

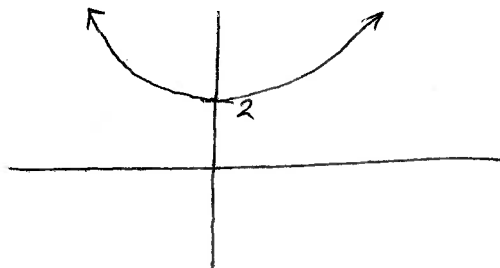
Sketch $f(x) = x^2$



Sketch $f(x) = \frac{1}{2}x^2$



Sketch $f(x) = \frac{1}{2}x^2 + 2$



Sketch $f(x) = \frac{1}{2}(x - 3)^2 + 2$

