Math 205 – Topics in Number Theory, Winter 2019

Topic: Characteristic p Iwasawa Theory and Applications

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The main goal of this course is to formulate and prove an equivariant main conjecture in Iwasawa theory for characteristic p global fields (a.k.a. function fields) and show how this result implies refinements of the classical Brumer-Stark and Coates-Sinnott conjectures in that setting.

The equivariant main conjecture in this setting relates in a very explicit manner a certain Galois equivariant ell-adic L-function to certain homological invariants (Fitting ideals) of the ell-adic realization of a certain 1-motive (a mixed Hodge structure) constructed by Deligne in the 1970s. In some sense, the equality predicted by this main conjecture "happens" at the top of the naïve cyclotomic tower of the function fields in question. We will show how via Iwasawa co-descent along this tower the main conjecture leads to very precise links between special values of global Artin L-functions at non-positive integers and the even Quillen K-groups of certain rings of integers of the base field, settling this way refinements of the Brumer-Stark and Coates-Sinnott conjectures.

The main bibliographical source for the material described above will be my joint paper with Cornelius Greither, listed as [1] below.

Time permitting, I may describe a much more general main conjecture in function fields, along Drinfeld cyclotomic towers, recently formulated and proved by Werner Bley and myself; also, I may describe very briefly how the results in [1] below were extended by Greither and myself to the (much more difficult) number field setting [2].

Bibliography:

[1] C. Greither and C. Popescu, *The Galois module structure of ell-adic realizations of Picard 1-motives and applications*, IMRN 2012, Issue 5, pp. 986-1036.

[2] C. Greither and C. Popescu, *An equivariant main conjecture in Iwasawa theory and applications,* Jour. of Alg. Geom. No. 24 (2015), pp. 629-692.

Background requirements: Math 200, Math 204 (knowledge of class field theory, some homological algebra, and basic group and Galois cohomology.) Some knowledge of ell-adic etale cohomology and Quillen K-theory is a plus, but not absolutely necessary.