

HW #6

6.1 #4

Considered the set of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in \mathbb{R}^2 with $x \geq y$.

with the usual vector addition and scalar multiplication.

Check the 10 axioms of a vector space:

Because we are using the usual vector addition and ~~an~~ scalar multiplication, the axioms that only involve $+$ and \cdot (and not V) are satisfied because \mathbb{R}^2 is a vector space.

So we only need to check axioms (1), (4), (5), and (6).

(1) If $\begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x' \\ y' \end{bmatrix}$ obey $x \geq y, x' \geq y'$, then $\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+x' \\ y+y' \end{bmatrix}$
obeys $x+x' \geq y+y'$. ✓

(4) The 0 vector is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$. Note $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is in the subset because $0 \geq 0$. ✓

(5) If $\begin{bmatrix} x \\ y \end{bmatrix}$ with $x \geq y$, then $-\begin{bmatrix} x \\ y \end{bmatrix}$ "should be" $\begin{bmatrix} -x \\ -y \end{bmatrix}$. However $x \geq y \Rightarrow -x \leq -y$, so whenever $x \neq y$, $\begin{bmatrix} -x \\ -y \end{bmatrix}$ is not in our subset. Axiom 5 FAILS.

(6) If $\begin{bmatrix} x \\ y \end{bmatrix}$ with $x \geq y$, then $c \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$. However $cx \not\geq cy$ if

$c \neq 1$ is negative and y is nonzero, so it's not in our subset. Axiom 6 FAILS.

6.1 #57 Is $h(x) = \sin(2x)$ in the span of $f(x) = \sin^2 x$
and $g(x) = \cos^2 x$?

If so, then

$$\sin(2x) = a_1 \sin^2 x + a_2 \cos^2 x.$$

We'll try to find the coefficients by evaluating the expressions
at different points.

$$\underline{x=0}: \quad \underbrace{\sin(2 \cdot 0)}_0 = a_1 \underbrace{\sin^2(0)}_0 + a_2 \underbrace{\cos^2(0)}_1$$

$$\text{So } 0 = a_2.$$

$$\underline{x = \pi/2}: \quad \underbrace{\sin(2 \cdot \pi/2)}_0 = a_1 \underbrace{\sin^2(\pi/2)}_1 + a_2 \underbrace{\cos^2(\pi/2)}_0$$

$$\text{So } 0 = a_1.$$

$$\text{Thus } \sin(2x) = 0.$$

However, this is obviously wrong (e.g. $\sin(2 \cdot (\pi/4)) = 1$),

so it must be that $\sin(2x)$ is not in the
span of $\sin^2 x$ and $\cos^2 x$.

6.1 #24 $V = \mathbb{R}^3$, $W = \left\{ \begin{bmatrix} a \\ 0 \\ a \end{bmatrix} \right\}$

To see if $W \subseteq V$ is a subspace, we use Theorem 6.2.

It suffices to check if W is closed under $+$ and \cdot .

closed under $+$: $\begin{bmatrix} a \\ 0 \\ a \end{bmatrix} + \begin{bmatrix} b \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} a+b \\ 0 \\ a+b \end{bmatrix} \in W \checkmark$

closed under \cdot : $c \begin{bmatrix} a \\ 0 \\ a \end{bmatrix} = \begin{bmatrix} ca \\ 0 \\ ca \end{bmatrix} \in W \checkmark$

So W is a subspace.

6.1 #45 $V = \mathcal{F} \leftarrow$ real-valued functions on \mathbb{R} .

$$W = \left\{ f \in \mathcal{F} \mid \lim_{x \rightarrow 0} f(x) = \infty \right\}.$$

Again use Theorem 6.2 to see if $W \subseteq V$ is a subspace.

closed under \cdot : We first check if W is closed under multiplication by 0. i.e. is $0 \cdot f = 0 \in W$?

$$\lim_{x \rightarrow 0} 0 = 0, \text{ so } 0 \notin W. \text{ Not closed under } \cdot.$$

Hence W is not a subspace.