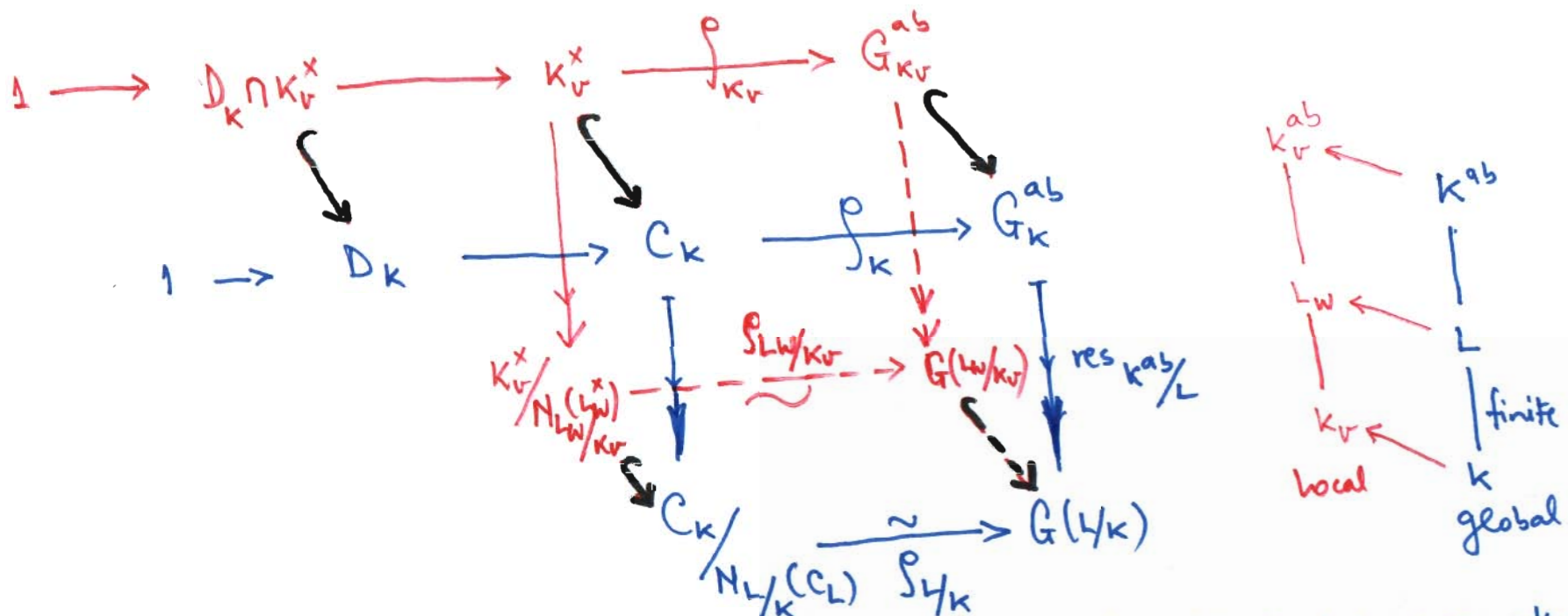


LOCAL/GLOBAL RECIPROcity MAPS

MATH 205 (Popescu)
Sp06 - F06 - Wi 07



- $\text{Im}(\rho_K)$ is dense in G_K^{ab} . If $\text{char}(K) = 0$, then $\text{Im}(\rho_K) = G_K^{ab}$. If $\text{char}(K) = p > 0$, then $\text{Im}(\rho_K) = \{ \sigma \in G_K^{ab} \mid \sigma(j) = j^{q_K^n}, \text{ for some } n \in \mathbb{Z}, \forall j \in \overline{\mathbb{F}_p}^{\times} \}$.
 - $\ker(\rho_K) = D_K = \bigcap_{L/K \text{ ab. finite}} N_{L/K}(C_L)$ is the connected component of 1 in C_K . If $\text{char}(K) = 0$, then $D_K = \{1\}$ (structure thm. due to Weil). If $\text{char}(K) = p > 0$, then $D_K = \{1\}$.
- $\text{Im}(\rho_{K_v})$ is dense in $G_{K_v}^{ab}$. If $K_v = \mathbb{R}$ or \mathbb{C} , then $\text{Im}(\rho_{K_v}) = G_{K_v}^{ab}$. If v is non-archimedean, then $\text{Im}(\rho_{K_v}) = \{ \sigma \in G_{K_v}^{ab} \mid \sigma|_{K_v^{\times}} = \sigma_v^n, \text{ for some } n \in \mathbb{Z} \}$.
 - $\ker(\rho_{K_v}) = D_K \cap K_v^{\times} = \text{the connected comp. of } 1 \text{ in } K_v^{\times} = \begin{cases} 1, & \text{if } v \text{ is non-archimedean} \\ \mathbb{C}^{\times}, & \text{if } K_v = \mathbb{C} \\ \mathbb{R}^+, & \text{if } K_v = \mathbb{R}. \end{cases}$