Math 100C, Spring 2012

Second Homework Assignment

Homework problems due Friday, April 20:

In text, Sec. 7.1, pp. 321–322: 1 (See Th. 7.1.3.), 9, 11, 14 (a_{11} must also be nonzero for the matrices in G). In addition for problem 14 prove that $G/N \cong \mathbb{R}^{\times}$.

Added Problems (These are also part of the homework.):

1. Let G be a p-group, i.e., G is a finite group wih $|G| = p^n$ for some prime number p and positive integer n. For this problem you will need to use the fact that if G is any p-group, then its center is nontrivial, i.e., $|Z(G)| > 1$. (See text, Th. 7.2.8, p. 328.)

(a) Prove that the p-group G has a normal subgroup N with $|N| = p$.

(b) Prove that if $|G| = p^n$ then G has normal subgroups $N_1, N_2, \dots, N_n$ such that $N_1 \subseteq N_2 \subseteq \dots \subseteq N_n$ and $|N_i| = p^i$ for each i.

2. Let G be a group (possibly infinite). Let N be a normal subgroup of G and H a subgroup of G with $N \subseteq H \subseteq G$. Suppose $|G\!:\!H| < \infty$. The goal of this problem is to prove that

$$\big|G/N\!:\!H/N\big| = |G\!:\!H|. \tag{$*$}$$

(a) Prove $(*)$ if $|G| < \infty$ by using Lagrange's Theorem.

(b) Prove $(*)$ if H is a normal subgroup of G using the Second Isomorphism Theorem.

(c) Now prove $(*)$ in general, assuming only that $|G\!:\!H| < \infty$. (Parts (a) and (b) won't help here.)