March 14, 2006

Course Announcement - Math 205, Spring 2006

LOCAL CLASS FIELD THEORY

The main goal of local class field theory is a precise description of the maximal abelian extension of an arbitrary local field. A field is called local if it is complete with respect to a discrete valuation and has a finite residue field. It turns out that a field is local if and only if it is either a finite extension of a ***p***-adic field ***Q_p*** or a field of power series in one variable with coefficients in a finite field ***F_q((X)).*** Consequently, local fields arise naturally as completions of global fields (i.e. number fields or function fields in one variable over finite fields) with respect to their non-Archimedean (discrete) valuations.

There are several approaches to local class field theory: 1) Hasse's approach (1930s) via the theory of central simple algebras and Brauer groups; 2) the Galois-cohomological approach developed in the 1950s by Artin, Hochschild, Nakayama, Tate and Weil; 3) the formal group approach, developed in the mid 1960s by Lubin and Tate; 4) the more modern and somewhat more direct approaches due to Hazewinkel (1970s) and Neukirch (1980s). Although I will comment on all these approaches, the course will revolve around a detailed description of a combination of Hazewinkel's and Neukirch's constructions, which will permit us to state and prove the main theorem (the local reciprocity law) relatively quickly.

Although I will not follow any text very closely, the recommended bibliography for this course is as follows.

1. *K. Iwasawa,* **Local Class Field Theory,** Oxford University Press (1986)
Note: this title is out of print. The UCSD bookstore is supposed to make copies and sell them at a reasonable price (with the publisher's permission).

2. *J.P. Serre,* **Local Fields,** Second Edition, Springer Verlag (1995)

3. *I. Fesenko, S. Vostokov,* **Local Fields and Their Extensions,** Second Edition, AMS Translations of Math. Monographs (2001)

Graduate and advanced undergraduate students are invited to attend. Knowledge of graduate level abstract algebra and basic algebraic number theory (e.g. global fields and their basic properties) will be very helpful.

Note: This is the first in a series of three one-quarter courses in Algebraic Number Theory – **Local Class Field Theory** (Spring 2006), **Global Class Field Theory** (Fall 2006), **Special Values of p-adic and global L-functions** (Winter 2007).

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