

# On elements of $\mathbf{a}$ -value 2 in Coxeter groups

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## Abstract

The  $\mathbf{a}$ -function on a Coxeter group  $W$  is a function  $\mathbf{a} : W \rightarrow \mathbb{N}$  defined by Lusztig which is intimately related to the partition of  $W$  into Kazhdan–Lusztig cells and to the representation theory of the Hecke algebra of  $W$ . It is known that the identity element of  $W$  is the only element with  $\mathbf{a}$ -value 0, while a non-identity element has  $\mathbf{a}$ -value 1 if and only if it has a unique reduced word. However, as its definition relies on the Kazhdan–Lusztig basis of the Hecke algebra, the  $\mathbf{a}$ -function is often difficult to compute for general elements.

In this talk we will focus on elements of  $\mathbf{a}$ -value 2, or  $\mathbf{a}$ -2 elements. We show that  $\mathbf{a}$ -2 elements are *fully commutative* in the sense of Stembridge, which allows us to associate to them certain posets called *heaps* and, in many cases, certain *generalized Temperley–Lieb diagrams*. Using heaps and Temperley–Lieb diagrams, we conjecture a combinatorial characterization of  $\mathbf{a}$ -2 elements, classify all Coxeter groups with finitely many  $\mathbf{a}$ -2 elements, and enumerate  $\mathbf{a}$ -2 elements for all groups from the classification. Joint with Richard Green.