Joints of Varieties

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Abstract

In 2010 Guth and Katz proved the following joints theorem: n lines in $\mathbb{R}^3$ form at most $O(n^{3/2})$ joints, where a joint is an intersection point of 3 non-coplanar lines. Their proof was one of the first applications of the polynomial method in incidence geometry. We generalize this result from lines to varieties of arbitrary dimensions. One special case of our result states that n planes in $\mathbb{F}^6$ (for any field $\mathbb{F}$) form at most $O(n^{3/2})$ joints, where a joint is an intersection point of 3 planes that do not all lie in a single hyperplane. Our results introduce new techniques for applying the polynomial method to higher-dimensional objects.

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