Clique and Berge Supersaturation for $K_{2,t}$

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Abstract

A famous conjecture of Erdős and Simonovits says that if $G$ is an $n$ vertex graph with much more than $(n, F)$ edges, then $G$ contains about as many copies of $F$ as the random graph of the same density. In this talk we show that several natural generalizations of this conjecture fails to be true. In particular, we show that for large $t$, there exist $n$ vertex graphs with $\Theta(kn^{3/2})$ triangles such that $G$ contains a total of $k^t n^{3/2+o(1)}$ copies of $K_{2,t}$ (with the random graph of the same triangle density containing $\Theta(k^{2t/3}n^2)$ copies), and we show that this bound is essentially best possible for $k \leq n^{1/2t}$. Our constructions rely on solving certain unbalanced bipartite Turán problems using random polynomial graphs. This is joint work with Quentin Dubroff, Benjamin Gunby, and Bhargav Narayanan.