Catalan functions and k-Schur functions

Professor Anna Pun
Drexel University

Abstract

Li-Chung Chen and Mark Haiman studied a family of symmetric functions called Catalan (symmetric) functions which are indexed by pairs consisting of a partition contained in the staircase $(n-1,...,1,0)$ (of which there are Catalan many) and a composition weight of length n. They include the Schur functions ,the Hall-Littlewood polynomials and their parabolic generalizations. They can be defined by a Demazure-operator formula, and are equal to GL-equivariant Euler characteristics of vector bundles on the flag variety by the Borel-Weil-Bott theorem. We have discovered various properties of Catalan functions, providing a new insight on the existing theorems and conjectures inspired by Macdonald positivity conjecture.

A key discovery in our work is an elegant set of ideals of roots that the associated Catalan functions are k-Schur functions and proved that graded k-Schur functions are G-equivariant Euler characteristics of vector bundles on the flag variety, settling a conjecture of Chen-Haiman. We exposed a new shift invariance property of the graded k-Schur functions and resolved the Schur positivity and k-branching conjectures by providing direct combinatorial formulas using strong marked tableaux. We conjectured that Catalan functions with a partition weight are k-Schur positive which strengthens the Schur positivity of Catalan function conjecture by Chen-Haiman and resolved the conjecture with positive combinatorial formulas in cases which capture and refine a variety of problems.

This is joint work with Jonah Blasiak, Jennifer Morse and Daniel Summers.