

Bollobás Set k -tuples

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Abstract

The Bollobás set pairs inequality is a fundamental result in extremal set theory. Given $\mathcal{A} = \{A_1, A_2, \dots, A_m\}$ and $\mathcal{B} = \{B_1, B_2, \dots, B_m\}$ families of finite sets, we say that $(\mathcal{A}, \mathcal{B})$ is a Bollobás set pair if we have that $A_i \cap B_j \neq \emptyset$ if and only if $i = j$. Given such a set pair, Bollobás (1965) showed

$$(1) \quad \sum_{i=1}^m \binom{|A_i \cup B_i|}{|A_i|}^{-1} \leq 1.$$

We examine suitable conditions on k -wise intersections from a k -tuple of set families for which an inequality in a similar form as (1) holds. Orlin (1977) connected the problem of covering a particular graph to Bollobás set pairs via Erdős, Goodman and Pósa's (1966) notion of set intersection graph. We connect Bollobás set k -tuples to a covering problem of a few particular k -uniform hypergraphs in a similar manner. We also provide random and explicit constructions of these Bollobás set k -tuples and discuss a few open problems. Joint work with Jacques Verstraete.