Unit-graphs and Special Unit-digraphs on Matrix Rings

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Abstract

In this talk we use the *unit-graphs* and the *special unit-digraphs* on matrix rings to show that every $n \times n$ nonzero matrix over $\mathbb{F}_q$ can be written as a sum of two SL_n-matrices when $n > 1$. We compute the eigenvalues of these graphs in terms of Kloosterman sums and study their spectral properties; and prove that if X is a subset of $\mathrm{Mat}_2(\mathbb{F}_q)$ with size $|X| > \frac{2q^3\sqrt{q}}{q-1}$, then X contains at least two distinct matrices whose difference has determinant α for any $\alpha \in \mathbb{F}_q^*$. Using this result we also prove a sum-product type result: if $A, B, C, D \subseteq \mathbb{F}_q$ satisfy $\sqrt[4]{|A||B||C||D|} = \Omega(q^{0.75})$ as $q \to \infty$, then $(A-B)(C-D)$ equals all of $\mathbb{F}_q$. In particular, if A is a subset of $\mathbb{F}_q$ with cardinality $|A| > \frac{3}{2}q^{\frac{3}{4}}$, then the subset $(A-A)(A-A)$ equals all of $\mathbb{F}_q$. We also recover a classical result: every element in any finite ring of odd order can be written as the sum of two units.