The $\mathbb{Z}_2$-genus of complete bipartite graphs

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Abstract

A drawing of a graph on a surface is *independently even* if every pair of nonadjacent edges in the drawing crosses an even number of times. The strong Hanani-Tutte theorem states that a graph admitting an independently even drawing in the plane must be planar.

The *genus* $g(G)$ of a graph G is the minimum g such that G has an embedding on the orientable surface M_g of genus g. The $\mathbb{Z}_2$*-genus* of a graph G, denoted $g_0(G)$, is the minimum g such that G has an independently even drawing on the orientable surface of genus g. Clearly, every graph G satisfies $g_0(G) \leq g(G)$, and the strong Hanani-Tutte theorem states that $g_0(G) = 0$ if and only if $g(G) = 0$. Although there exist graphs G for which the values of $g(G)$ and $g_0(G)$ differ, several recent results suggest that these graph parameters are closely related. We provide further evidence of their similarity.

For complete bipartite graphs $K_{n,m}$ with $n \geq 3$, we prove the following:

$$g_0(K_{n,m}) \geq \left\lceil \frac{1}{2} \left(\left\lceil \frac{n-2)(m-2)}{2} \right\rceil - (n-3) \right) \right\rceil$$

The value of $g(K_{n,m})$ was determined by Ringel in 1965, and equals $\left\lceil \frac{(n-2)(m-2)}{4} \right\rceil$.

Joint work with J. Kynčl.