A general incidence bound in high dimensions

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Abstract

In this talk, I will present a general upper bound for the number of incidences with k-dimensional varieties in $\mathbb{R}^d$ such that their incidence graph does not contain $K_{s,t}$ for fixed positive integers s, t, k, d (where $s, t > 1$ and $k < d$). The leading term of this new bound generalizes previous bounds for the special cases of $k = 1, k = d - 1$, and $k = d/2$. Moreover, we find lower bounds showing that this leading term is tight (up to sub-polynomial factors) in various cases. To prove our incidence bounds, we define k/d as the dimension ratio of an incidence problem. This ratio provides an intuitive approach for deriving incidence bounds and isolating the main difficulties in each proof. We use this approach to derive an incidence bound for hyperplanes in complex spaces. This is joint work with Adam Sheffer.