Math 21D (Driver) Practice Final Exam

This does not represent the full final exam material, mostly just material which was not covered on the first two midterms.

The following figure is used in the next two problems.

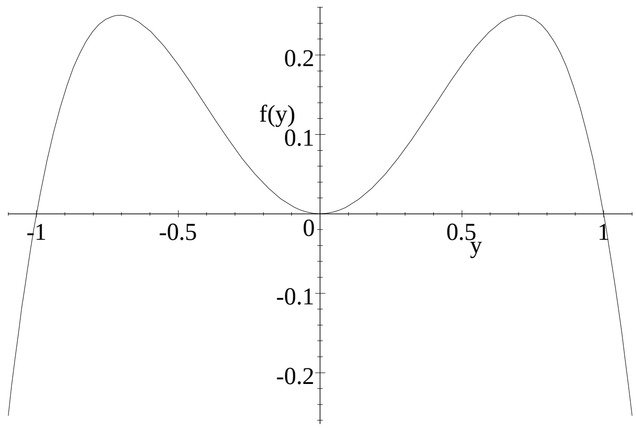

Figure 1. Plot of $f(y)$ versus y.

The zeros of the function f are located at $y = -1$, 0, and 1.

1. Sketch the direction fields at $y = -1.5,\ -1,\ -0.5,\ 0,\ 0.5,\ 1,$ and 1.5 for the differential equation

$$y'(t) = f(y(t))$$

where f is the function graphed in Figure 1 above. (The coordinate axis of your picture should be labeled by y and t.)

2. Find and classify (as stable, unstable, or semi-stable) the equilibrium points for the differential equation

$$y'(t) = f(y(t))$$

where f is the function graphed in Figure 1 above.

3. Find **one** solution to the differential equation: $y'' + 2y' = 3 + 4\sin(2t)$.

4. Find a fundamental set of solutions to the differential equation

$$x^2 y'' + 2xy' + 4y = 0. \tag{1}$$

5. Determine which of the following two equation has a regular singular point at $x = 0$ **and** for that equation write down the indicial equation.

$$xy'' + 2y' + (1 + x^2)y = 0. \tag{A}$$

$$x^2y'' + 3y' + \sin(x)y = 0. \tag{B}$$

6. Compute the Wronskian of the pair of functions $y_1(x) = \sin(x)$ and $y_2(x) = x\cos(x)$. Are the functions linearly independent?

7. Use the method of variation of parameters to find one solution to the differential equation:

$$x^2y'' - 2y = 3x^2 - 1 \text{ for } x > 0.$$

8. Let $y(t)$ solve the initial value problem:

$$y'' - 4y' + 4y = e^t \cos 3t \quad \text{with } y(0) = 1 \text{ and } y'(0) = 3.$$

   Find $Y(s) \equiv \mathcal{L}(y)(s)$ — the Laplace transform of y.

9. Find the continuous function $f(t)$ such that

$$\mathcal{L}(f)(s) = \frac{s^3 + 4s + 4}{(s^2 + 4)s^2}.$$

   (**Note:** You will be given the necessary portions of Table 6.2.1 on p. 300 of Boyce and Diprima to do the Laplace transform problems. So do not bother to copy the table to your cheat sheet.)

10. Set $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Find a recurrence relation on the coefficients $\{a_n\}$ so that $y(x)$ will solve the differential equation:

$$(1 - x)y'' + y = 0.$$

11. Hint: one of the word problems from your homework assignments **will** be on the final.