Math 180C (Driver) Final Take Home Problems
Due June 11, 2008 at the Final

Instructions: These problems are due at the final (although early submissions are fine as well). Please work alone on these problems. You may use the book, lecture notes, and your class notes as reference. If you have questions about the problems, please ask them in class on Wednesday June 4 or Friday June 6 so that everyone gets the same information. Please put your name and section number on your solutions and label them as **Final Take Home Problems.** You should do all of the problems below. Please explain your work so as to get **full credit.** **GOOD LUCK!!**

1 Part I: Take home problems

These problems relate to the **cumulative process** considered on page 451-452 of the book. Let $\{(X_i, Y_i)\}_{i=1}^{\infty}$ i.i.d. sequence of random vectors, with $X_i \geq 0$, $W_0 = 0$, $W_n = X_1 + \cdots + X_n$, and $N(t) = \#\{n : W_n \leq t\}$ be the renewal process associated to $\{X_i\}$. The Y_i are random variables representing an amount earned or lost at the end of the i^{th} renewal period. The **cumulative process (or reward process)** is

$$R(t) := \sum_{i=1}^{N(t)+1} Y_i,$$

represents the amount earned or lost at end of the renewal period containing t. For all problems on this page, it is assumed that there is a probability density, $f(x)$, such that

$$F(t) = P(X_i \leq t) = \int_0^t f(x)\, dx.$$

(In solving Problems 1 – 3, you might find it useful to look at Examples 10.29 and 10.34 of the lecture notes.)

Pr. 1 (10 Points)**.** *Find expressions for* $\mathbb{E}[R(t)\,|X_1 = x]$ *when a)* $t < x$, *and b)* $t \geq x$.

Pr. 2 (10 Points)**.** *Let* $g(t) = \mathbb{E}[R(t)]$. *Use your results from the previous problems to show* g *satisfies the renewal equation,* $g = \mathbb{E}Y_1 + g * F$, *i.e.*

$$g(t) = \mathbb{E}Y_1 + \int_0^t g(t-x)\, dF(x) \text{ for all } t \geq 0. \tag{1}$$

Pr. 3 (10 Points)**.** *Write the solution to Eq. (1) in terms of* $\mathbb{E}Y_1$ *and* $M(t) = \mathbb{E}N(t)$ *and use this to prove the "**Reward Renewal Theorem,**"*

$$\lim_{t\to\infty} \frac{\mathbb{E}R(t)}{t} = \frac{\mathbb{E}Y_1}{\mathbb{E}X_1}.$$

Remark. *In words, the Reward Renewal Theorem states; the long time rate of reward is the expected reward during one renewal period divided by the expected duration of a renewal period.*

Pr. 4 (10 Points)**.** *A policeman spends his entire day on the lookout for speeders. The policeman cruises on average approximately 10 minutes before stopping a car for some offense. Of the cars he stops, 90% of the drivers are given speeding tickets with an \$80 fine. It takes the policeman an average of 5 minutes to write such a ticket. The other 10% of the stops are for more serious offenses, leading to an average fine of \$300. These more serious charges take an average of 30 minutes to process. Making use of the reward renewal theorem to find the long run rate of money brought in by fines.*