7. Study Guide for Math 120A Midterm 1 (Friday October 17, 2003)

(1) $\mathbb{C} := \{z = x + iy : x, y \in \mathbb{R}\}$ with $i^2 = -1$ and $\bar{z} = x - iy$. The complex numbers behave much like the real numbers. In particular the quadratic formula holds.

(2) $|z| = \sqrt{x^2 + y^2} = \sqrt{z\bar{z}}$, $|zw| = |z|\,|w|$, $|z + w| \le |z| + |w|$, $\operatorname{Re} z = \frac{z+\bar{z}}{2}$, $\operatorname{Im} z = \frac{z-\bar{z}}{2i}$, $|\operatorname{Re} z| \le |z|$ and $|\operatorname{Im} z| \le |z|$. We also have $\overline{zw} = \bar{z}\bar{w}$ and $\overline{z+w} = \bar{z} + \bar{w}$ and $z^{-1} = \frac{\bar{z}}{|z|^2}$.

(3) $\{z : |z - z_0| = \rho\}$ is a circle of radius ρ centered at z_0.
$\{z : |z - z_0| < \rho\}$ is the open disk of radius ρ centered at z_0.
$\{z : |z - z_0| \ge \rho\}$ is every thing outside of the open disk of radius ρ centered at z_0.

(4) $e^z = e^x (\cos y + i \sin y)$, every $z = |z|\, e^{i\theta}$.

(5) $\arg(z) = \left\{\theta \in \mathbb{R} : z = |z|\, e^{i\theta}\right\}$ and $Arg\,(z) = \theta$ if $-\pi < \theta \le \pi$ and $z = |z|\, e^{i\theta}$. Notice that $z = |z|\, e^{i \arg(z)}$

(6) $z^{1/n} = \sqrt[n]{|z|} e^{i \frac{\arg(z)}{n}}$.

(7) $\lim_{z \to z_0} f(z) = L$. Usual limit rules hold from real variables.

(8) Mapping properties of simple complex functions

(9) The definition of complex differentiable $f(z)$. Examples, $p(z)$, e^z, $e^{p(z)}$, $1/z$, $1/p(z)$ etc.

(10) Key points of e^z are is $\frac{d}{dz} e^z = e^z$ and $e^z e^w = e^{z+w}$.

(11) All of the usual derivative formulas hold, in particular product, sum, and chain rules:

$$\frac{d}{dz} f(g(z)) = f'(g(z))\, g'(z)$$

and

$$\frac{d}{dt} f(z(t)) = f'(z(t))\, \dot{z}(t).$$

(12) $\operatorname{Re} z$, $\operatorname{Im} z$, $\bar{z}$, are nice functions from the real - variables point of view but are **not** complex differentiable.

(13) Integration:

$$\int_a^b z(t)\, dt := \int_a^b x(t)\, dt + i \int_a^b y(t)\, dt.$$

All of the usual integration rules hold, like the fundamental theorem of calculus, linearity and integration by parts.