Math 20E A00 Fall 2021: Homework 4

Instructor: Brian Tran

Due Thursday, October 28, 11:59 pm.

Remark. Problems written as "Exercise X.Y.Z" are from the textbook, section X.Y exercise Z. For example, Exercise 5.3.8 denotes exercise 8 of section 5.3. For problems referring to a figure, find the question in the textbook for the corresponding figure. Make sure to show all of your work and steps; credit will not be given for just stating an answer.

Although the problem may not ask for it explicitly, it is always helpful to sketch the domain (when applicable).

Problem 1 Exercise 7.3.2

Find an equation for the plane tangent to the given surface at the specified point:

$$x = u^2 - v^2,\ y = u + v,\ z = u^2 + 4v,$$

at $(x, y, z) = (-1/4, 1/2, 2)$.

Problem 2 Exercise 7.3.8

Match the following parametrizations to the surfaces shown in figures (i) through (iv).

(a) $\Phi(u, v) = (u\cos v, u\sin v, 4 - u\cos v - u\sin v); u \in [0, 1], v \in [0, 2\pi]$

(b) $\Phi(u, v) = (u\cos v, u\sin v, 4 - u^2)$,

(c) $\Phi(u, v) = (u, v, \frac{1}{3}(12 - 8u - 3v))$,

(d) $\Phi(u, v) = ((u^2 + 6u + 11)\cos v, u, (u^2 + 6u + 11)\sin v)$.

Problem 3 Exercise 7.3.11

Find an expression for a unit vector normal to the surface

$$\Phi(u, v) = (\sin v, u, \cos v),$$

at the image of Φ at a point (u, v) where $v \in [0, 2\pi], u \in [-1, 3]$. Identify this surface.

Remark: Note that the problem asks for the **unit** normal vector (recall that a unit vector is a vector of magnitude 1); so, compute the normal vector and divide by its magnitude to get a unit normal vector.

Problem 4 Exercise 7.4.5(c)

Let $\Phi(u, v) = (e^u \cos v, e^u \sin v, v)$ be a mapping from $D = [0, 1] \times [0, \pi]$ in the uv plane onto a surface $S = \Phi(D)$.

(c) Find the area of $\Phi(D)$.

Problem 5 Exercise 7.4.6

Find the area of the surface defined by $z = xy$ and $x^2 + y^2 \leq 2$.

Problem 6 Exercise 7.4.9

Let $\Phi(u, v) = (u - v, u + v, uv)$ and let D be the unit disc in the uv plane, $D = \{(u, v) : u^2 + v^2 \leq 1\}$. Find the area of the surface $\Phi(D)$.

Problem 7 Exercise 7.4.10

Find the area of the portion of the unit sphere that is cut out by the cone $z \geq \sqrt{x^2 + y^2}$.

Hint: The domain of one of the parameters (θ, ϕ) become restricted when we cut out the sphere by a cone.

Problem 8 Exercise 7.5.2

Evaluate the integral of the function $f(x, y, z) = z + 6$ over the surface $S = \Phi(D)$ given by

$$\Phi(u, v) = \left(u, \frac{v}{3}, v\right), \ (u, v) \in D = [0, 2] \times [0, 3].$$

Problem 9 Exercise 7.5.4

Evaluate the integral

$$\iint_S (x + z) dS,$$

where S is the part of the cylinder $y^2 + z^2 = 4$ with $x \in [0, 5]$.

Problem 10 Exercise 7.5.6

Evaluate the integral

$$\iint_S (x^2 z + y^2 z) dS,$$

where S is the part of the plane $z = 4 + x + y$ that lies inside the cylinder $x^2 + y^2 = 4$.