Math 20D Summer Session 1 2022: Homework 1

Instructor: Brian Tran

Due Wednesday, July 6, 11:59 pm.

Remark. Problems written as "Exercise X.Y.Z" are from the textbook, section X.Y exercise Z. For example, Exercise 1.2.4 denotes exercise 4 of section 1.2. For problems referring to a figure, find the question in the textbook for the corresponding figure. Make sure to show all of your work and steps; credit will not be given for just stating an answer.

Problem 1 Exercise 1.2.4

Determine whether the given function is a solution to the given differential equation.

$$x = 2\cos(t) - 3\sin(t), \quad x'' + x = 0.$$

Problem 2 Exercise 1.2.10

Determine whether the given relation is an implicit solution to the given differential equation. Assume that the relation defines y implicitly as a function of x and use implicit differentiation.

$$y - \ln(y) = x^2 + 1, \quad \frac{dy}{dx} = \frac{2xy}{y-1}.$$

Problem 3 Exercise 2.2.10

Find the general solution for the given differential equation

$$\frac{dy}{dx} = \frac{x}{y^2\sqrt{1+x}}.$$

Problem 4 Exercise 2.2.18

Solve the IVP

$$y' = x^3(1-y),$$
$$y(0) = 3.$$

Problem 5 Exercise 2.2.24

Solve the IVP

$$\frac{dy}{dx} = 8x^3 e^{-2y},$$
$$y(1) = 0.$$

Problem 6 Exercise 2.3.10

Obtain the general solution to the equation

$$x\frac{dy}{dx} + 2y = x^{-3}.$$

Problem 7 Exercise 2.3.18

Solve the IVP

$$\frac{dy}{dx} + 4y - e^{-x} = 0,$$
$$y(0) = \frac{4}{3}.$$

Problem 8 Exercise 2.3.20

Solve the IVP

$$\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x,$$
$$y(1) = 1.$$

Problem 9 First-order Systems

In the first week of class, we focused on first-order differential equations. These are the most important class of differential equations from a theoretical perspective, because every n^{th}-order differential equation can be transformed into a system of n first-order differential equations.

Consider the general form of a second-order differential equation

$$\frac{d^2x}{dt^2} = f\left(t, x, \frac{dx}{dt}\right).$$

By defining a new variable $v = dx/dt$, show that this second-order equation can be transformed into a system of two first-order differential equations of the form

$$\frac{dx}{dt} = g(t, x, v),$$
$$\frac{dv}{dt} = h(t, x, v).$$

Express g and h in terms of t, x, v, f.

(Optional) Show that a general n^{th}-order differential equation

$$\frac{d^nx}{dt^n} = f\left(t, x, \frac{dx}{dt}, \ldots, \frac{d^{n-1}x}{dt^{n-1}}\right)$$

can be transformed into a system of n first-order differential equations.

Problem 10 Exercise 2.4.10

Determine whether the equation is exact. If it is, then solve it.

$$(2x + y)dx + (x - 2y)dy = 0.$$

Problem 11 Exercise 2.4.22

Solve the IVP

$$\begin{aligned}(ye^{xy} - 1/y)dx + (xe^{xy} + x/y^2)dy = 0, \\ y(1) = 1.\end{aligned}$$

Problem 12 Exercise 2.4.24

Solve the IVP

$$\begin{aligned}(e^t x + 1)dt + (e^t - 1)dx = 0, \\ x(1) = 1.\end{aligned}$$

Problem 13 Exercise 2.5.8

Find the general solution for the equation

$$(3x^2 + y)dx + (x^2 y - x)dy = 0.$$

Problem 14 Exercise 2.5.14

Find an integrating factor of the form $x^n y^m$ and solve the equation

$$(12 + 5xy)dx + (6xy^{-1} + 3x^2)dy = 0.$$

Problem 15 Exercise 4.2.6

Find a general solution to the given differential equation.

$$y'' - 5y' + 6y = 0.$$

Problem 16 Exercise 4.2.16

Solve the IVP

$$\begin{aligned}y'' - 4y' - 5y = 0, \\ y(-1) = 3, \\ y'(-1) = 9.\end{aligned}$$