# Math 20A Summer Bridge 2021: Homework 5

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Due Wednesday, September 8, 11:59 pm

**Remark.** Problems written as "Exercise X.Y.Z" are from the textbook, section X.Y exercise Z. For example, Exercise 2.2.8 denotes Exercise 8 of section 2.2. For problems referring to a figure, find the question in the textbook for the corresponding figure.

**Remark.** You can apply any theorem or rule (as long as the problem does not say explicitly to not use that theorem or rule), but make sure to show that the assumptions of said theorem or rule apply.

## Problem 1 Integrating Odd and Even Functions

**(a)** We say that $f : \mathbb{R} \to \mathbb{R}$ is an odd function if $f(x) = -f(-x)$ for any $x$ (an example of such a function is $f(x) = x$). Assume that $f$ is integrable on the region $[-a, a]$ where $a > 0$. Graphically, explain why

$$\int_{-a}^{a} f(x)dx = 0.$$

**(b)** We say that $f : \mathbb{R} \to \mathbb{R}$ is an even function if $f(x) = f(-x)$ for any $x$ (an example of such a function is $f(x) = x^2$). Assume that $f$ is integrable on the region $[-a, a]$ where $a > 0$. Graphically, explain why

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx.$$

## Problem 2 Practice Integrating Piecewise Functions

Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = |2x - 6|$. Using geometry, compute

$$\int_{2}^{6} f(x)dx.$$

**Hint:** Use that $|t|$ equals $t$ for $t > 0$ and $-t$ for $t < 0$ to give a piecewise definition of $f(x)$ (when is $2x - 6$ positive and negative?). Then, split the integral from 2 to 6 into the regions where $f$ is piecewsie defined. Subsequently, use geometry to evaluate the integral in each region.

## Problem 3 Exercise 5.3.4

Find all antiderivatives of $f$ (that is, find the indefinite integral $\int f(x)\,dx$) and check your answer by differentiating.

$$f(x) = 9x + 15x^{-2}.$$

## Problem 4 Exercise 5.3.17

Evaluate the indefinite integral (stating which rules you are using).

$$\int (z^{-4/5} - z^{2/3} + z^{5/4})dz.$$

## Problem 5 Exercise 5.3.28

Evaluate the indefinite integral (stating which rules you are using).

$$\int (\theta + \sec^2\theta)d\theta.$$

## Problem 6 Exercise 5.3.44

Verify by differentiation:

$$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + C, \text{ for } n \neq -1.$$

## Problem 7 Exercise 5.4.10

Evaluate using FTC I:

$$\int_{-2}^{2} (10x^9 + 3x^5)dx.$$

## Problem 8 Practice with the FTC I

Evaluate using FTC I:

$$\int_0^2 \left(3\cos(\pi x) - e^{2x+4} + \frac{1}{x+1}\right)dx.$$

**Hint:** Use linearity to break up the integral into three separate integrals and then apply FTC I to each integral. For the exponential term, note $e^{2x+4} = e^4 e^{2x}$.

## Problem 9 Exercise 5.4.55

Evaluate $\int_{-2}^{3} f(x)dx$, where

$$f(x) = \begin{cases} 12 - x^2, & x \leq 2, \\ x^3, & x > 2. \end{cases}$$

## Problem 10 Practice with the Comparison Theorem

Recall, in lecture, we showed that $\sin(x) \leq x$ for any $x \geq 0$. Using the comparison theorem, prove the inequality (for $u \geq 0$)

$$1 - \cos(u) \leq \frac{1}{2}u^2.$$

**Hint:** Recall that the comparison theorem says that if $f(x) \leq g(x)$ on an interval $[a, b]$ and if $f$ and $g$ are integrable on $[a, b]$, then one has

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx.$$

Use the comparison theorem with $f(x) = \sin(x), g(x) = x, a = 0, b = u$ (**note** our choices of $f$ and $g$ are integrable on $[0, u]$ since they are continuous) to prove the desired inequality.

## Problem 11 More practice integrating piecewise functions

Evaluate $\int_0^3 f(x)dx$ where

$$f(x) = \begin{cases} x^2, & 0 \leq x < 1 \\ e^x, & 1 \leq x < 2, \\ 2\cos(\pi x), & 2 \leq x \leq 3. \end{cases}$$

## Problem 12 Exercise 5.5.6

Compute the area function $A(x)$ of $f(x)$ with lower limit $a$. Then, verify the FTC II relationship by checking that $A'(x)$ equals $f(x)$.

$$f(x) = 1 - x + \cos(x),\ a = 0.$$

**Hint:** Recall that area function $A$ of a function $f$ with lower limit $a$ is given by

$$A(x) = \int_a^x f(t)dt.$$

## Problem 13 Exercise 5.5.28

Calculate the derivative (using FTC II)

$$\frac{d}{ds}\int_{-2}^{s} \tan\left(\frac{1}{1+u^2}\right) du.$$

## Problem 14 Practice with the (chain rule version of) FTC II

Calculate the derivative

$$\frac{d}{dx}\int_{x^2}^{e^x} \ln(t+1)dt.$$

**Hint:** Using the properties of the definite integral, we can write

$$\int_{x^2}^{e^x} \ln(t+1)dt = \int_{x^2}^{0} \ln(t+1)dt + \int_{0}^{e^x} \ln(t+1)dt = -\int_{0}^{x^2} \ln(t+1)dt + \int_{0}^{e^x} \ln(t+1)dt.$$

Subsequently, differentiate the two integrals on the right hand side using the chain rule version of the FTC II.