Math 20A Summer Bridge 2021: Homework 3

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Due Tuesday, August 24, 11:59 pm

Remark. Problems written as "Exercise X.Y.Z" are from the textbook, section X.Y exercise Z. For example, Exercise 2.2.8 denotes Exercise 8 of section 2.2. For problems referring to a figure, find the question in the textbook for the corresponding figure.

Remark. You can apply any theorem or rule (as long as the problem does not say explicitly to not use that theorem or rule), but make sure to show that the assumptions of said theorem or rule apply.

Problem 1 Practice with Differentiating Trig. Functions

Compute the derivative df/dx where

$$f(x) = x\cos(x^2+1)\tan(x).$$

Where is the derivative defined (hint: where is $1/\cos(x)$ defined)?

Problem 2 Exercise 3.6.30

Find an equation of the tangent line to f at the point specified,

$$f(x) = \csc(x) - \cot(x),\ x = \pi/4.$$

Problem 3 Deriving the Derivative of a Trig. Function

Derive the formula

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x),$$

by applying the quotient rule to the function $\sec(x) = 1/\cos(x)$. Where is the derivative of $\sec(x)$ defined?

Problem 4 Exercise 3.8.18

Calculate the derivative with respect to x of the other variables appearing in the equation,

$$\sqrt{x+s} = \frac{1}{x} + \frac{1}{s}.$$

(That is, compute the derivative of the above equation with respect to x, viewing s as a function of x, and solve for ds/dx)

Problem 5 Exercise 3.8.66

Show that no point on the implicitly defined curve $x^2 - 3xy + y^2 = 1$ has a horizontal tangent line.

Hint: First, take the derivative of the above equation with respect to x which will give you an equation involving x, y, and dy/dx. Then, in this equation, assume $dy/dx = 0$ (since this is precisely when one has a horizontal tangent line). This will give you a relation between x and y. Solve this relation for y in terms of x and plug this into the original equation $x^2 - 3xy + y^2 = 1$ and show that there are no solutions to the resulting equation, which shows that the assumption that there is a point such that $dy/dx = 0$ is false; i.e., there is no horizontal tangent line.

Problem 6 Using implicit differentiation to derive the derivative of an inverse

Using implicit differentiation, derive the derivative of the inverse tangent function:

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{x^2+1}.$$

Hint: To do this, let $y = \tan^{-1}(x)$ which gives $\tan(y) = x$. Implicitly differentiate this equation and solve for the derivative. This will give dy/dx in terms of x and y, but we want dy/dx just in terms of x. In order to replace y for x, $\tan(y) = x$ tells us to consider a right triangle with one of the acute angles being y such that the ratio of the opposite over the adjacent is x. Draw such a triangle (taking for simplicity the opposite to be x and the adjacent to be 1). This triangle then allows one to compute any trig. function of y in terms of x (such as $\cos(y), \sec(y), \sin(y)$, etc...)

Problem 7 Practice Differentiating Logarithms

Consider the function

$$f(x) = \ln\Big((\ln x)^3\Big).$$

Where is this function defined? (To answer this, recall that the logarithm is defined for positive inputs. Thus, in order for $\ln((\ln x)^3)$ to be defined, we need $(\ln x)^3 > 0$; for what x values is this true?)

Subsequently, compute df/dx.

Problem 8 Practice with Logarithmic Differentiation

Using logarithmic differentiation, compute the derivative df/dx of

$$f(x) = x^{2^x}$$

(to be clear, since the font above is quite small, $f(x)$ is given by x raised to the 2^x). Recall that logarithmic differentiation gives the derivative of f via the formula

$$f'(x) = f(x)\frac{d}{dx}\ln(f(x)).$$

Problem 9 Exercise 3.10.12

Refer to a 5 m ladder sliding down a wall, as in Figures 5 and 6. The variable h is the height of the ladder's top at time t and x is the distance from the wall to the ladder's bottom.

Suppose that the top is sliding down the wall at a rate of 1.2 m/s. Calculate dx/dt at $t = 2s$ (remember, with related rates problems, be careful about signs: positive derivatives indicate increasing along the defined axis, negative derivatives indicate decrease along the defined axis).

Problem 10 Exercise 3.10.22

A laser pointer is placed on a platform that rotates at a rate of 20 revolutions per minute. The beam hits a wall 8 m away, producing a dot of light that moves horizontally along the wall. Let θ be the angle between the beam and the line through the searchlight perpendicular to the wall (Figure 11). How fast is this dot moving when $\theta = \pi/6$?

Problem 11 Exercise 4.1.14

Using linear approximation, estimate $\Delta f = f(b) - f(a)$. Use this to estimate $f(b)$ and find the error using a calculator.

$$f(x) = x^{1/4},\ a = 16,\ b = 16.5$$

(That is, use linear approximation to estimate $f(b)$ given that we know $f(a)$, and then compare your estimate to a better answer that one would get from a calculator).

Problem 12 Exercise 4.1.28

Find the linearization $L(x)$ of $f(x)$ about $x = a$ and then use it to to compute the approximation of $f(b)$ by $L(b) \approx f(b)$, where

$$f(x) = e^x \ln(x),\ a = 1,\ b = 1.02$$

(note: you can leave your answers in terms of e).

Problem 13 Exercise 4.1.42

The resistance R of a copper wire at temperature $T = 20°C$ is $R = 15$ ohms. Using linear approximation, estimate the resistance at $T = 22°C$, assuming that $(dR/dT)|_{T=20°C} = 0.06$ ohms/$°C$.

Problem 14 Exercise 4.2.16

Find all critical points of the function $R : \mathbb{R} \to \mathbb{R}$ given by

$$R(\theta) = \cos(\theta) + \sin^2(\theta).$$

Problem 15 Exercise 4.2.32

Find the minimum and maximum values of the function on the given interval by comparing values at the critical points and endpoints,

$$f(x) = x^3 - 6x^2 + 8,\ [-1, 6]$$

(recall the theorem discussed in class and in the textbook: a continuous function on a closed interval achieves its minimum and maximum, and the points where those minimum and maximum occur are either at the endpoints of the interval or at the critical points).