Math 18 Summer Session 1 2023: Homework 1

Instructor: Brian Tran

Due Wednesday, July 12, 11:59 pm.

Remark. Problems written as "Exercise X.Y.Z" are from the textbook, section X.Y exercise Z. For example, Exercise 1.2.4 denotes exercise 4 of section 1.2. For problems referring to a figure, find the question in the textbook for the corresponding figure. Make sure to show all of your work and steps; credit will not be given for just stating an answer.

Problem 1 Exercise 1.1.8

The augmented matrix of a linear system has been reduced by row operations to the form shown. Continue the appropriate row operations and describe the solution set of the original system.

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}.$$

Hint: Continue the row operations until the augmented matrix is in reduced row echelon form (see Section 1.2). This will allow you to explicitly write the solution set of the linear system.

Problem 2 Exercise 1.2.12

Find the general solution of the system whose augmented matrix is given by

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}.$$

Problem 3 Exercise 1.2.22

Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix}.$$

Hint: From Theorem 2 of Section 1.2, we have a characterization of when an augmented matrix corresponds to a consistent linear system. Attempt to convert the above matrix into a row echelon form. For what value(s) of h will this theorem tell us that the matrix corresponds to a consistent linear system?

Problem 4 Exercise 1.3.12

Determine if $\vec{b}$ is a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

$$\vec{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \ \vec{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \ \vec{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}.$$

Problem 5 Exercise 1.3.18

Let

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}, \ \vec{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}.$$

For what value(s) of h is $\vec{y}$ in the plane spanned by $\vec{v}_1$ and $\vec{v}_2$?

Hint: This is similar to Problem 3, using the equivalence between linear systems and vector equations discussed in Section 1.3.

Problem 6 Exerise 1.4.12

Write the augmented matrix for the linear system that corresponds to the matrix equation $A\vec{x} = \vec{b}$. Then, solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

Problem 7 Exercise 1.5.10

Describe all solutions of $A\vec{x} = \vec{0}$ in parametric vector form, where A is row equivalent to the given matrix.

$$A = \begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \end{bmatrix}.$$

Problem 8 Exercise 1.5.48

Construct a 3×3 nonzero matrix A such that the vector $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is a solution of $A\vec{x} = \vec{0}$.

Problem 9 Exercise 1.7.2

Determine if the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \ \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \ \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}.$$

Problem 10 Exercise 1.7.33

How many pivot columns must a 7×5 matrix have if its columns are linearly independent? Why?

Problem 11 Exercise 1.7.34

How many pivot columns must a 5×7 matrix have if its columns span $\mathbb{R}^5$? Why?

Problem 12 Exercise 1.8.10

Find all $\vec{x} \in \mathbb{R}^4$ that are mapped into the zero vector by the linear transformation $\vec{x} \mapsto A\vec{x}$ corresponding to the matrix

$$A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}.$$

Problem 13 Exercise 1.8.12

Let $\vec{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}$ and let A be the matrix in Exercise 1.8.10 (Problem 12). Is $\vec{b}$ in the range of the linear transformation $x \mapsto A\vec{x}$? Why or why not?

Problem 14 Exercise 1.8.18

The figure shows vectors $\vec{u}, \vec{v}, \vec{w}$, along with the images $T(\vec{u})$ and $T(\vec{v})$ under the action of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$. Copy this figure carefully, and draw the image $T(\vec{w})$ as accurately as possible.

Hint: First, write $\vec{w}$ as a linear combination of $\vec{u}$ and $\vec{v}$. Then, use the linearity of T to express $T(\vec{w})$ in terms of $T(\vec{u})$ and $T(\vec{v})$.

Problem 15 Exercise 1.9.6

Assume that T is a linear transformation. Find the matrix corresponding to T, where $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a horizontal shear transformation that leaves $\vec{e}_1$ unchanged and maps $\vec{e}_2 \mapsto \vec{e}_2 + 3\vec{e}_1$.

Problem 16 Exercise 1.9.37

Describe the possible echelons forms of the matrix corresponding to the linear transformation T, where $T : \mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one (injective).

Problem 17 Exercise 1.9.38

Describe the possible echelons forms of the matrix corresponding to the linear transformation T, where $T : \mathbb{R}^4 \to \mathbb{R}^3$ is onto (surjective).