Lie Group Variational Collision Integrators for a Class of Hybrid Systems

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A hybrid system is a dynamical system that exhibits both continuous and discrete dynamic behavior. The state of a hybrid system changes either continuously by the flow described by a differential equation or discretely following some jump conditions. A canonical example of a hybrid system is the bouncing ball, imagined as a point-mass, under the influence of gravity. In this talk, we explore the solutions and algorithms to the extensions of this example in 3-dimension, where the body of interest is rigid and convex in general and the plane may be tilted. In particular, the solutions utilize the theory of nonsmooth Lagrangian mechanics [1] to derive the differential equations and jump conditions, which heavily depend on the collision detection function. The proposed algorithm called Lie group variational collision integrator (LGVCI) is developed using the combination of techniques and knowledge from variational collision integrators in [1] and Lie group variational integrators in [2]. Furthermore, we also developed a sensible and practical regularization (by analysis and applying ϵ-rounding on signed distance functions in [3] and [4]) for collision response for convex rigid bodies with corners, and this completely avoids the need for nonsmooth convex analysis,and computations of tangent and normal cones. We have extensive numerical experiments and animations from our algorithm demonstrating that LGCVI are symplectic-momentum preserving and have long-time, near energy conservation.

This is a joined work with Professor Melvin Leok, and we are looking to apply and extend this work in the fields of control & optimal control theory and robotics, especially in the realm of bipedal robots. There will be further discussions on these topics in the section of future directions of the talk.

References

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