Department of Mathematics,
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Center for Computational Mathematics Seminar

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Discrete Multisymplectic and Variational Principles of the FEM in Lagrangian Field Theory

Abstract:

The variational principle of Lagrangian PDE is encoded geometrically in a multisymplectic structure. This multisymplectic structure gives rise to a covariant formulation of Noether's theorem and conservation of multisymplecticity, a spacetime generalization of the symplecticity of Lagrangian mechanics. Multisymplecticity has many important physical implications, such as reciprocity in electromagnetism and conservation of wave action, a key ingredient in the stability analysis of wave propagation problems. Furthermore, upon introducing a foliation of spacetime, multisymplecticity can be reformulated as symplecticity on an infinite-dimensional phase space. Consequently, understanding how these multisymplectic structures are affected under discretization is an important aspect to the numerical integration of this class of PDE.

In this talk, after discussing the multisymplectic formulation of Lagrangian PDE, we discuss how the multisymplectic structure is affected under discretization of the variational principle via the finite element method. We show how choices of discretization which preserve functional and geometric relationships of the underlying function spaces naturally give rise to discrete analogs of the continuum multisymplectic structures. In particular, we discuss how cochain projections and group-equivariant projections from the continuum function spaces to the finite element function spaces induce discrete analogs of the variational, multisymplectic, and Noether-theoretic structures.

This is joint work with Melvin Leok.

Tuesday, December 1, 2020
11:00 AM
Zoom Meeting ID: 926 7798 0955

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