Department of Mathematics,
University of California San Diego

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Center for Computational Mathematics Seminar

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Multisymplectic Hamiltonian Variational Integrators

Abstract:

The multisymplectic structure of Lagrangian and Hamiltonian PDEs is a covariant generalization of the field-theoretic symplectic structure and encodes many important physical conservation laws. Multisymplectic integrators are a class of numerical methods which, at the discrete level, preserve the multisymplectic structure of a Lagrangian or Hamiltonian field theory. By preserving this structure at the discrete level, a multisymplectic integrator admits discrete analogs of the conservation laws encoded by multisymplecticity. Such methods have been used, for example, for structure-preserving modeling of nonlinear wave phenomena and for stable discretizations of plasma physics problems. There have also been recent investigations into the application of discrete multisymplectic structures for lattice quantum field theory.

Traditionally, such multisymplectic integrators have been constructed from the variational perspective in the Lagrangian framework, or from directly discretizing the equations of motion in the Hamiltonian framework and subsequently determining the conditions for the discretization method to be multisymplectic in an ad hoc manner. In this talk, after discussing the necessary background material, I will discuss a systematic framework for constructing multisymplectic Hamiltonian integrators variationally utilizing the notion of a Type II generating functional. This framework only requires a choice of finite-dimensional function space and quadrature, so it is applicable to unstructured meshes, whereas traditional Hamiltonian multisymplectic integrators require rectangular meshes. As an application of this framework, I will derive the class of multisymplectic partitioned Runge–Kutta methods and show that, in this framework, discretizing via a tensor product partitioned Runge–Kutta expansion in spacetime is well-defined if and only if the partitioned Runge–Kutta methods are symplectic in space and time. This is joint work with Prof. Melvin Leok.

Time permitting, I will discuss future research directions, such as applications to lattice quantum field theory.

Tuesday, April 13, 2021
11:00 AM
Zoom ID 939 3177 8552

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