Type II Hamiltonian Lie Group Variational Integrators for Geometric Adjoint Sensitivity Analysis

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We present our construction of continuous and discrete Type II variational principles for Hamiltonian systems on cotangent bundles of Lie groups, which allows for Type II boundary conditions, i.e., fixed initial position and terminal momenta boundary conditions. The motivation for these boundary conditions arises from the adjoint sensitivity method, which is ubiquitous in dynamically-constrained optimization and optimal control problems. Traditionally, such Type II variational principles are only defined locally. However, for dynamics on the cotangent bundle of a Lie group, left-trivialization allows us to define this variational principle globally. Our discrete variational principle leads to an intrinsic, symplectic, and momentum-preserving integrator for Lie group Hamiltonian systems that allows for Type II boundary conditions and maximally degenerate Hamiltonians. We show how this method can be used to exactly compute sensitivities for optimization problems subject to dynamics on a Lie group. We conclude with numerical examples of optimal control problems on SO(3).

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