

Read page 107 through page 132. Read page 121 #21 for the “golden section.”

1. Find the canonical continued fraction of $\sqrt{6}$. That is, find the integers such that

$$\sqrt{6} = n_0 + \frac{1}{n_1 + \frac{1}{n_2 + \frac{1}{\ddots}}}, n_0 \geq 0, n_1 > 0, n_2 > 0, \dots$$

2. Find the canonical continued fraction of the golden ratio, $\frac{\sqrt{5} + 1}{2}$.

3. Look at page 120 #11. In studying the canonical continued fraction of $\sqrt{3}$, we can observe that $\{\sqrt{3} - 1\} = \frac{1}{1 + \frac{1}{2 + \{\sqrt{3} - 1\}}}$. Further, we can find increasing approximations

defined by $a_0 = 0$, and $a_{k+1} = \frac{1}{1 + \frac{1}{2 + a_k}}$ for $k = 0, 1, 2, \dots$. In particular,

$$\sqrt{3} > 1 + a_5 = \frac{265}{153}, \text{ the lower estimate used by Archimedes.}$$

Similarly, we can find decreasing approximations by $b_0 = 1$ and $b_{k+1} = \frac{1}{1 + \frac{1}{2 + b_k}}$ for

$k = 0, 1, 2, \dots$. Show that $\sqrt{3} < 1 + b_5 = \frac{1351}{780}$, the upper estimate used by Archimedes to estimate π .

4. Page 120 #15. (As part of your general education, you should know one or more proofs of the Pythagorean Theorem.)

5. Page 121 #17

6. That $\sqrt{2}$ is irrational is shown on page 112 of our text. (You might start reading on page 111 to have the history in mind.) This proof can be extended in various ways. In particular, it can be extended to show that $\sqrt{5}$ is irrational. I suggest that you think of “odd” and “even” in the proof on page 112 as relating to the remainder when dividing an integer by 2. To extend the proof, consider the possible remainders when an integer is divided by 5. Namely, the possible values are 0, 1, 2, 3 and 4. For example, if the remainder is 4, then $m = 5k + 4$. In this case, $m^2 = (5k + 4)^2 = 5^2 k^2 + 2 \cdot 5k \cdot 4 + 4^2 = 5[5k^2 + 2 \cdot k \cdot 4 + 3] + 1$ and m^2 is not divisible by 5.

Consider the other possibilities and show that $\sqrt{5}$ is irrational.

Homework VI

Due Thursday, February 11, 2010 in section.

Read about the three construction problems studied, but not solved in antiquity, pages 122 through 141. Know the names and the statements of those problems. Read pages 143 through 171.

1. Page 121 #18. Look at page 121 #19 and 20.
2. Page 129 #1.
3. Page 129 #2.
4. Page 130 #4.
5. Page 131 #7.
6. Page 139 #2

Look at <http://aleph0.clarku.edu/~djoyce/java/elements/toc.html> the symbol after aleph is “zero.” From the Clark University website the following is an outline of the contents of Euclid's Elements

Book I. The fundamentals of geometry: theories of triangles, parallels, and area.
Definitions (23) Postulates (5) Common Notions (5) Propositions (48)

Book II. Geometric algebra.

Book III. Theory of circles.

Book IV. Constructions for inscribed and circumscribed figures.

Book V. Theory of abstract proportions.

Book VI. Similar figures and proportions in geometry.

Book VII. Fundamentals of number theory.

Book VIII. Continued proportions in number theory.

Book IX. Number theory.

Book X. Classification of incommensurables. Propositions (115)

Book XI. Solid geometry.

Book XII. Measurement of figures.

Book XIII. Regular solids. Propositions (18)