Discrepancy in modular arithmetic progressions

Yunkun Zhou

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Abstract

Celebrated theorems of Roth and of Matoušek and Spencer together show that the discrepancy of arithmetic progressions in the first n positive integers is $\Theta(n^{1/4})$. We study the analogous problem in the $\mathbb{Z}_n$ setting. We asymptotically determine the logarithm of the discrepancy of arithmetic progressions in $\mathbb{Z}_n$ for all positive integer n. We further determine up to a constant factor the discrepancy of arithmetic progressions in $\mathbb{Z}_n$ for many n. For example, if $n = p^k$ is a prime power, then the discrepancy of arithmetic progressions in $\mathbb{Z}_n$ is $\Theta(n^{1/3+r_k/(6k)})$, where $r_k \in \{0, 1, 2\}$ is the remainder when k is divided by 3. This solves a problem of Hebbinghaus and Srivastav. Joint work with Jacob Fox and Max Wenqiang Xu.