

Triangles in the integer grid $[n] \times [n]$

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I will show some results, and numerous open problems about triangles spanned by the points of the integer grid $[n] \times [n]$. Here are a few examples:

- What is the largest set $S \subset [n] \times [n]$ not containing an isosceles triangle? This is an old question of Erdős, but no reasonable upper or lower bounds are known.
- How many colours do we need to colour the points of $[n] \times [n]$ such that there is no monochromatic triple spanning an isosceles right triangle? If we restrict our attention to axis-parallel right triangles, then we have some bounds. We need at least $\log \log(n)$ colours, and there are colourings using $n^{o(1)}$ colours to avoid such monochromatic triangles. If we drop the axis-parallel condition, do we need at least n^c colours for some $c > 0$ constant?
- What is the maximum number of triples in $[n] \times [n]$ spanning triangles with the same perimeter?

There are more easy-to-ask and (probably) hard-to-answer questions about triples of the integer grid $[n] \times [n]$. Many of them seem to be good research problems for graduate students.