Large monochromatic components in a complete hypergraph

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Abstract

Gyárfás famously showed that in every r-coloring of the edges of the complete graph K_n , there is a monochromatic connected component with at least $\frac{n}{r-1}$ vertices. A recent line of study by Conlon, Tyomkyn, and the speaker addresses the analogous question about monochromatic connected components with many *edges*. In this talk, we discuss a generalization of these questions for k-uniform hypergraphs. Over a wide range of extensions of the definition of connectivity to higher uniformities, we provide both upper and lower bounds for the size of the largest monochromatic component (in terms of the number of vertices, edges, or more generally vertex s-sets contained within edges) that are tight up to a factor of 1 + o(1) as the number of colors grows. Along the way, we highlight an interesting connection with the existence of certain combinatorial designs.

This talk is based on joint work with Lyuben Lichev.