Fixing a hole

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Abstract

We show that any finite $S \subset \mathbb{R}^d$ in general position has arbitrarily large supersets $T \supseteq S$ in general position with the property that T contains no empty convex polytope, or hole, with C_d points, where C_d is an integer that depends only on the dimension d. The key step in our proof, which may be of independent interest, is to show that there are arbitrarily small perturbations of the set of lattice points $[n]^d$ with no large holes.