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Colloquium Talk

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Transport equations in random matrices and non-commutative probability

Abstract:

We investigate the analogs of optimal transport theory in the setting of multivariable asymptotic random matrix theory. Asymptotic random matrix theory concerns the behavior of randomly chosen $N \times N$ matrices in the limit as $N \to \infty$. For several random matrices $X_1^{(N)}, \ldots, X_d^{(N)}$, one can study the asymptotic behavior of expressions like $(1/N)\backslash\mathbf{Tr}(X_{i_1} \ldots X_{i_k})$, and the appropriate limiting object is a non-commutative probability space, that is, a von Neumann algebra A of "random variables" together with an expectation map $E : A \to \mathbb{C}$, analogous to the expected trace of a random matrix. Meanwhile, optimal transport theory asks for the most efficient way to rearrange one distribution of mass μ on $\mathbb{R}^d$ into another such distribution ν. Such a scheme is often given by transporting the mass at point x to poir $f(y)$, for a smooth function $f : \mathbb{R}^d \to \mathbb{R}^d$.

Optimal transport is more challenging to make sense of in the non-commutative setting because, unlik classical probability theory, there are many non-isomorphic atomless non-commutative probabilit spaces, and in fact, space of non-commutative probability distributions fails basic separability and finite dimensional approximation properties that one is used to in classical probability. So there is often n possibility of transporting given non-commutative probability distribution μ to ν by some map f nonetheless, for the relaxed problem of optimal couplings, we can recover a non-commutative analog c the Monge-Kantorovich duality characterizing optimal couplings. Furthermore, in the regime of conve free Gibbs laws (an analog of smooth log-concave probability measures on $\mathbb{R}^d$), non-commutativ transport can be achieved by non-commutative smooth functions obtained as solutions to differentia equations much like the classical case. Moreover, the non-commutative analog of triangula transformations of measures led to new insight into the structure of the underlying von Neuman algebras.

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