A PROBLEM OF E. VAN DOUWEN AND A. MILLER

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We say that two functions $f, g \in {}^\omega\omega$ are eventually different (e.d.) iff $|f \cap g| < \omega$. In [M:P], A. Miller stated a problem first asked by E. van Douwen:

Question 1. *CH implies there exists $\mathcal{A} \subseteq {}^\omega\omega$ which is maximal with respect to eventually different (e.d.) functions which is also maximal with respect to infinite partial functions. Is there always such an $\mathcal{A}$? What is $\mathfrak{a}_{\mathrm{m}}$, the cardinality of the smallest such $\mathcal{A}$?*

We can partially answer Quesstion 1 by proving the following theorems.

Theorem 2.(MA)**.** *There exists an $\mathcal{A} \subseteq {}^\omega\omega$, which is maximal with respect to e.d. functions which is also maximal with respect to infinite partial functions, and $\mathfrak{a}_{\mathrm{m}} = 2^\omega$.*

Theorem 3. *Let $M \models (ZFC + \neg CH)$. Let $\kappa \in M$ be a cardinal such that $\omega_1 \leq \kappa < 2^\omega = \lambda$. Then there exists a c.c.c. notion of forcing $\mathbb{P}$ such that the following statements holds in $M^{\mathbb{P}}$.*

(1) $2^\omega = \lambda$.
(2) *There exists a m.e.d. function set $A^\star \subseteq {}^\omega\omega$ of cardinality κ with respects to both eventually different functions and infinite partial functions.*

Theorem 4. *Let $M \models (ZFC + CH)$. There is a m.e.d. family $\mathcal{A} \in M$ of size ω_1 in M such that for any Cohen generic G over M, $\mathcal{A}$ remains a m.e.d. family in $M[G]$. Here $\mathcal{A}$ is maximal with respect to eventually different functions and $\mathcal{A}$ is also maximal with respect to infinite partial functions.*

Definition 5. Let $\mathfrak{a}$ be the least infinite λ such that there exists a maximal almost disjoint (m.a.d.) family $\mathcal{F} \subset \wp(\omega)$ of size λ.

Theorem 6. *It is consistent with ZFC that $\mathfrak{a}_{\mathrm{m}} = \omega_2$ and $\mathfrak{a} = \omega_1$.*

References

[M:P] A. Miller, *Some interesting problems (updated at 15th Oct, 1996)*, Set Theory of the Reals (ed. Haim Judah) Israel Mathematical Conference Proceedings, vol 6, 1993, pp. 645–654, American Math Society.

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