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The First Order Theories of Dedekind Algebras

A Dedekind Algebra is an ordered pair (B, h) where B is a non-empty set and h is a similarity transformation on B. Each Dedekind algebra can be decomposed into a family of disjoint, countable subalgebras called the configuration of the Dedekind algebra. There are $\aleph_0$ isomorphism types of configurations. Each Dedekind algebra is associated with a cardinal-valued function on omega called its configuration signature. The configuration signature of a Dedekind algebra counts the number of configurations in the decomposition of the algebra in each isomorphism type.

The configuration signature of a Dedekind algebra encodes the structure of that algebra in the sense that two Dedekind algebras are isomorphic provided their configuration signatures are identical. Configuration signatures are used to establish various results in the first-order model theory of Dedekind algebras. These include categoricity results for the first order theories of Dedekind algebras and existence and uniqueness results for homogeneous, universal and saturated Dedekind algebras. Fundamental to these results is a condition on configuration signatures which is necessary and sufficient for elementary equivalence.