SATURATION PROPERTIES IN THE STRUCTURE OF COMPUTABLY ENUMERABLE DEGREES

DEJIA WANG

ABSTRACT. We show that given any finite partially ordered sets $P \subseteq Q_i$ with the least and greatest elements such that $\{Q_i : i \in \omega\}$ is a recursive sequence, if each Q_i satisfies Slaman-Soare's second non-extension condition, then there is an embedding e of P into the structure of computably enumerable (c.e.) degrees $\mathcal{R} = (\mathbf{R}, \leq, \mathbf{0}, \mathbf{0}')$ which cannot be extended to any embedding of Q_i into $\mathcal{R}$ for all $i \in \omega$. Such embedding e is said with *saturation* properties (also called with *strong embedding* properties). This result strengthens Slaman-Soare's second non-embedding theorem. It is a partial result of the decidability of the Π_2-theory of c.e. degrees. In addition, we will give a full characterization of embeddability of finite distributive lattices into the c.e. degrees with saturation properties, namely, a finite distributive lattice can be embedded into c.e. degrees with saturation properties iff it is a d.d.c. (disjoint diamond chain) except the diamond.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN - MADISON, WI 53706, USA
E-mail address: dwang@math.wisc.edu