Complexity of some substructural logics

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We prove two complexity results for propositional linear logic with the unrestricted weakening rule, and an extension of it.

Theorem 1 *(1) The decision problem for* **LLW** *is exponential space hard with respect to log-lin reducibility; (2) There is a constant* $c > 1$ *so that any Turing machine solving the decision problem for* **LLW** *must use space* c^n *for infinitely many inputs of length* n*. (3) The decision problem for* **LLW** *can be solved in doubly exponential space.*

The upper bound in part (3) improves on the upper bound derivable from Kopylov's original decision procedure [2], which is non-primitive-recursive.

Let **ELLW** be the sequent calculus of Extended Linear Logic of Kanovich [1], with the rule of weakening added. This calculus admits cut elimination, but does not satisfy the subformula property.

Theorem 2 *There is no primitive recursive decision procedure for* **ELLW***.*

References

[1] Max I. Kanovich. Linear logic as a logic of computations. *Annals of Pure and Appliced Logic*, 67:183–212, 1994.

[2] A.P. Kopylov. Decidability of linear affine logic. In *Proceedings of the 10th Annual IEEE Symposium on Logic in Computer Science*. IEEE Computer Society Press, 1995.