Computable dimension, degree spectra and expansions by constants

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The *computable dimension* of a structure $\mathcal{A}$ is the maximal cardinality of a set of computable structures $\mathcal{A}_i$ each isomorphic to $\mathcal{A}$ but pairwise not computably isomorphic. The *degree spectrum* of a relation R on a computable structure $\mathcal{A}$ is the set of the Turing degrees of all images of R under isomorphisms from $\mathcal{A}$ to a computable structure $\mathcal{B}$.

This talk will discuss recent results on constructing computable structures $\mathcal{A}$ and relations R on $\mathcal{A}$ while simultaneously controlling both the computable dimension of $\mathcal{A}$ and the degree spectrum of R. The results represent work of Denis Hirschfeldt, Bakhadyr Khoussainov and the speaker. They answer questions from [2] and [4]. The methods developed by Hirschfeldt [3] to solve these problems also provide the tools to settle a problem left open in [1] about the possible effects of expansions of a computably categorical structure $\mathcal{A}$ by constants on its computable dimension. All of the phenomena discussed occur in some common algebraic structres such as nilpotent groups, rings and even integral domains.

[1] P. Cholak, S. S. Goncharov, B. Khoussainov and R. A. Shore, Computably Categorical Structures and Expansions by Constants, *J. Symbolic Logic*, to appear.

[2] S. S. Goncharov and B. Khoussainov, Degree Spectra of Decidable Relations, *Dokl. Akadem. Nauk SSSR* **352** (1997), 301-303.

[3] D. Hirschfeldt, Ph.D. Dissertation, Cornell University, 1999.

[4] B. Khoussainov and R. A. Shore, Computable Isomorphisms, Degree Spectra of Relations and Scott Families, *Annals of Pure and Applied Logic* **93** (1998), 153-193.