ASYMPTOTICALLY DEFINABLE STRUCTURES

HANS SCHOUTENS

Many geomtrically meaningful sets are clearly not first order definable: for instance, the collection of prime ideals of an affine ring; the set of (isomorphism classes) of finitely generated modules over an affine ring, etc., are not first order definable over some field K. However, we can slice up these sets, such that each slice becomes definable. The basic idea is that we look at a slice consisting of data which can be described with a fixed number of polynomials of a fixed degree and the encoding is then given by a tuple listing all the coefficients of the polynomials involved. Sets of this form will be termed *asymptotically definable*. In an asymptotically definable structure, one can make first order statements about its elements and constructions applied to its elements. The prerequisite is that such a construction should be *bounded* in the following sense: if we start with an object described by polynomials of degree at most d, then the result after applying the construction is again an object described by polynomials of degree at most d', where d' only depends on d and not on the objects nor the base field.

We briefly discuss two applications of this technique: transfer of results from one characteristic to the other via the Lefschetz Principle and uniform constructibility results of certain numerical invariants. Among the former we will discuss the Bass Conjecture, the Zariski-Lipman Conjecture and the New Intersection Theorem; among the latter we will discuss Betti and Bass numbers, Cohen-Macaulay and Gorenstein loci, and Hilbert series.

MATHEMATICS DEPARTMENT
WESLEYAN UNIVERSITY
MIDDLETOWN, CT 06459 (US)
E-mail address: hschoutens@wesleyan.edu

Typeset by $\mathcal{AMS}$-TEX