Nonmonotonic Model Culmination

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Abstract A tree computing basis to nonmonotonic logic characterizes a model theory for default logic and entailment. A positive forcing basis to nonmonotonic logic allows us to define a standard model theory for nonmonotonic logic. Specific theorems characterize a proof theory and model theory for nonmonotonic logic by positive forcing[Nourani 1982]. A completeness theorem is proved on nonmonotonic logic by positive forcing. Applications to model discovery and new computing areas are presented in brief proving a completeness theorem on abductive model diagrams.

1. Positive Forcing Contains Default Logic

A default theory consists of a set A of sentences of first order logic together with a set of rules of inference called "defaults." Defaults have the form p(x1,...,xn): q(x1,...,xn) $\rightarrow$ r(x1,...,xn), where p,q, and r are first order formulas.

Definition We say that a formula g(x1,...,xn) is provable by default iff it can be deduced by the above default rule schemata and a given set of axioms. Let a default extension of a theory T(P) be T(P) U {r}, where r is a formula proved by default using the closure of the axioms of T(P) under the rules of inference of T(P) as the axiom set. Further, we say that a formula r is provable by default from a theory T(P) if r can be proved by default from any default extension of T(P). []

The notion of consistency in our formulation is that of consistency of a formula relative to an initial model characterizing the AI world. Thus if $\varphi(x) = t$ is consistent with the standrad model, then φ can be applied as a default.

Theorem 1 The formulas provable by default are only what is +forced. []

2.New Applications

[Poole 1988]'s theorist is a proof procedure for abductive reasoning based on [Loveland 1978]'s MESON system. Abduction involves constructing hypotheses, where each hypothesis is a set of atomic literals Δi; such that when some particular theory T is augmented with Δi, it entails the set of goal literals G, i.e. T $\cup$ Δi$\models$ G. Δi must be a subset of a set of ground atomic abducibles A. It addition we must ensure T $\cup$ Δi is consistent under $\models$. The set of all possible hypotheses is Δ= {Δi}. More general abductions are defined by applying more complex formulas as abducibles A. First order existential formuals can form the abducibles. According to Poole an explanation problem can be stated formally as: given Facts $\mathcal{F}$: consistent formulas, known to be true, Defaults $\triangle$: possible hypotheses, that we accept as part of an explanation and Observations $\mathcal{G}$: which are to be explained, An observation g $\in \mathcal{G}$ is explainable, if there exits a ground hypotheses $\mathcal{D} \subseteq \triangle$ such that: 1. $\mathcal{F} \bigcup \mathcal{D} \models$ g ; 2. $\mathcal{F} \bigcup \mathcal{D}$ is consistent. We have applied certain diagrams to abductive models since 1994 to model partial deduction and cumulative defaults. A **predictive diagram** for a theory T is a diagram D[M], where M is a model for T, and for any formula q in M, either the function f: q $\rightarrow$ {0,1} is defined, or there exists a formula p in D[M], such that T U {p} proves q; or that T proves q by minimal abduction. A **generalized predictive diagram**, is a predictive diagram with D[M] defined by a specific set of functions.

A specific to Poole's logic might be as follows.

Theorem 2 A set of first order observations $\mathcal{G}$ is explainable iff there exists an predictive diagram for the logical consequences to $\mathcal{G}$.[]

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