GENERALIZED LEVY-SHOENFIELD ABSOLUTENESS

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Let P be a partial order and κ a cardinal. Let $MA_\kappa(P)$ be the statement that for each $p \in P$ and each collection $\mathcal{D}$ of cardinality $\leq \kappa$ of subsets of P which are dense below p, there is a $|\mathcal{D}|$ generic $G \subseteq P$ such that $p \in G$. Let $A \subseteq \kappa$ and let $\phi(x)$ be a Σ_1 formula. Suppose $V \models MA_\kappa(P)$ and $V[G]$ is a forcing extension of V via the partial order P. Then

$$V \models \phi(A) \quad \Leftrightarrow \quad V[G] \models \phi(A).$$