Abstract: $0^\sharp$ and Elementary End Extensions of V_κ

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In this paper we consider the question of existence of elementary end extensions of models of the form (V_κ, ε). The interest in the structure of well-founded elementary end extensions of models of set theory goes back to the results of Keisler Morley and Silver. Villaveces [3] studied the structure of the partial order of elementary end extensions. A good overview of the subject is given in [3].

In this paper we prove the following :

Theorem 1.1 *If $0^\sharp$ exists then for every cardinal κ there is an inner model M such that*

$$M \models \text{“}(V_\kappa, \in) \quad \textit{has no well-founded elementary end extension}\text{”}.$$

In particular weak compactness is never downwards absolute, once we have $0^\sharp$ in the universe. On the other hand we will prove that any cardinal above $\aleph_1$ is Mahlo in any strict inner model of $L[0^\sharp]$. The proof of theorem 1.1 uses ideas of Kunen [1] for constructing a Suslin tree and of M. Stanley [2] for constructing generic objects for reverse Easton iterations inside $L[0^\sharp]$, together with a simple coding of the generic Suslin tree.

References

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[3] A. Villaveces. Height of models of ZFC and the existence of end elementary extensions. Preprint, 1996.