STABILITY IN FINITE MODELS

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We assume that τ is a finite similarity type, k is greater or equal to the arity of τ and in any case ≥ 2, $\mathcal{L} = L^k$, T is a complete $\mathcal{L}$-theory and

$$\mathbf{K} = \{\mathcal{A} |\ \mathcal{A} \models T \text{ is a finite } \tau\text{-model}\}.$$

In addition we assume that $\{\mathcal{A}/\cong\ |\ \mathcal{A} \in \mathbf{K}\}$ is infinite and that the following amalgamation property holds: Let $\mathcal{A}$ and $\mathcal{B}$ be $\mathbf{K}$-models (i.e. $\in \mathbf{K}$) and $t_{\mathcal{A}}(\mathcal{A} \cap \mathcal{B}, \emptyset) = t_{\mathcal{B}}(\mathcal{A} \cap \mathcal{B}, \emptyset)$. Then there is a $\mathbf{K}$-model $\mathcal{C}$ and a function $f : \mathcal{A} \cup \mathcal{B} \to \mathcal{C}$ such that $f \restriction \mathcal{A}$ and $f \restriction \mathcal{B}$ are $\mathcal{L}$-elementary.

We define a notion of stability for $\mathbf{K}$ and create a theory of forking under the assumption that $\mathbf{K}$ is stable. Finally, we apply the theory to model constructions, e.g. we prove, for all $n \in \mathbb{N}$, the existence of n-saturated $\mathbf{K}$-models by assuming that forking behaves nicely.