ON BOUNDING SIMPLE SETS INSIDE $\mathfrak{M}$-DEGREES

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ABSTRACT. We study nontrivial upper bounds with respect to m-reducibility for simple sets both as natural classes and as individual sets. The results along the former line are: (1) No m-incomplete computably enumerable set is an upper bound for all simple sets. (2) No effectively simple set is an upper bound for all strongly effectively simple sets; in addition, for every effectively simple set A there is a strongly effectively simple and c-complete (hypersimple) set B such that $B \not\leq_m A$. (3) If $\mathbb{C}$ is a Σ_3 class of computably enumerable sets containing only simple sets and cofinite sets, then the ideal $\mathbb{I}$ generated by $\mathbb{C}$ has an exact pair of simple sets and every set in $\mathbb{C}$ is 1-reducible to any one half of this exact pair; in particular, the ideal generated by the (strongly) effectively simple sets and cofinite sets has an exact pair of simple sets B and C, and every (strongly) effectively simple set is 1-reducible to B and to C. The results along the latter line are: (1) Below some noncomputable computably enumerable set there exists no simple set; in addition, this noncomputable computably enumerable set could be chosen to be p-complete. (2) Below every c-complete computably enumerable set there exists a simple set.

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