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Maximality vs. Extendability: A Zermellian-Structuralist View of Some Recent Results of McGee on Set-Theoretic Determinateness

In an interesting recent paper [1], Vann McGee has proved a "Categoricity Theorem" to the effect that any two models, having the same universe of discourse, of second order ZFC set theory with urelements satisfying the additional axiom that the urelements form a set are such that their pure sets are isomorphic. A particular case of this asserts that any two models of this theory "in which the first-order variables range over everything have isomorphic pure sets". An apparent virtue of this is that we can appeal to this theory to support the determinateness in truth-value of arbitrary, unbounded sentences of pure set theory, much as we can appeal to second-order Dedekind-Peano arithmetic to support the determinateness of arbitrary sentences of the language of that theory.

Two interrelated problems with this are raised and discussed. The first is that it conflicts with the open-endedness and extendability of universes of discourse for set theory, as articulated forcefully by Zermelo in his famous paper of 1930 [2]. Indeed, Zermelo's vision is shown here to be perfectly compatible formally with McGee's results. It is, however, useful toward this end to introduce modal concepts to avoid conflicts with second-order logical comprehension principles [3]. The second problem concerns the "non-mathematicality" and even circularity in invoking unrestricted quanification ("absolutely everything") for the purpose of establishing determinateness. Here it is granted that we do use unrestricted quantification in our ordinary discourse, but it is also urged that it be treated as open-ended and context-dependent, in line with the Zermellian view rather than the argument for full determinateness. Finally, we emphasize that the determinateness of the Continuum Hypothesis and indeed of any statement concerning accessible sets is not in question, and is sustained on both views.

References

[1] Vann McGee, "How We Learn Mathematical Language", *The Philosophical Review*, 106 1(1997), 35-68.
[2] Ernst Zermelo, "Über Grenzzahlen und Mengenbereiche: neue Untersuchungen über die Grundlagen der Mengenlehre", *Fundamenta Mathematica*, 16(1930), 29-47.
[33] See Geoffry Hellman, *Mathematics without Numbers: Towards a Modal-Structural Interpretation* (Oxford University Press, 1989), Ch. 2.