NORMAN DANNER AND DANIEL LEIVANT. *Stratified Polymorphism and Primitive Recursion.*
Department of Mathematics, Indiana University, Bloomington, IN 47405, USA.
E-mail: ndanner@indiana.edu
Department of Computer Science, Indiana University, Bloomington, IN 47405, USA.
E-mail: leivant@cs.indiana.edu

In [2], one of the authors introduced the system of finitely stratified polymorphism and characterized the numeric functions representable in it as $\mathcal{E}_4$, the fourth level of the Grzegorczyk hierarchy (the super-elementary functions). Finitely stratified polymorphism is a restriction of the polymorphic λ-calculus in which types are assigned finite levels, and type quantification is restricted by the levels in such a way that breaks the circularity inherent in full polymorphism. The relationship between stratified polymorphism and full polymorphism bears some similarity to that between ramified or predicative second-order logic (as in, e.g., [1] and [3]) and classical second-order logic.

Here we extend finitely stratified polymorphism to transfinite levels through ω^ω. Introduction of limit ordinal levels is handled by a highly uniform and finitely presentable inference similar to the Limit Generalization rule of [1]. We then generalize the result of [2] and show that the functions representable in the calculus using levels $< \omega^k$ consist of precisely $\mathcal{E}_{k+3}$ in the Grzegorczyk hierarchy; in particular, the full calculus captures exactly the primitive recursive functions.

References

[1] Solomon Feferman. Systems of predicative analysis. *J. Symbolic Logic*, 29(1):1–30, 1964.

[2] Daniel Leivant. Finitely stratified polymorphism. *Inform. and Comput.*, 93(1):93–113, 1991.

[3] Kurt Schütte. *Proof Theory.* Springer-Verlag, Berlin, 1977. Translated from the revised German edition by J. N. Crossley, Grundlehren der Mathematischen Wissenschaften, Band 225.